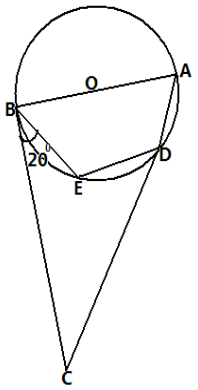
**O'LEVEL MATHEMATICS ONLINE SEMINAR 9TH May - 24TH MAY 2013.**

1. (a) Given that $3^x - 5 = 0$, **determine** without using mathematical tables or calculator the value of $9^{x+2}$

(b) In the figure below, O is the center of the circle. DC and BC are tangents to the circle at D and B respectively. AOB is a straight line $\angle EBC = 20^0$, $\angle OAD = 30^0$.




**Calculate;** giving reasons

a) $\angle$ DBE (b) $\angle$BED

© $\angle$CDB (d) $\angle$BCB

(e) $\angle$OEB

2. A certain number of people agreed to contribute to buy novels worth sh. 1200. Five of them pulled out and the others agreed to contribute an extra Sh. 10 each. Their contribution brought novels worth sh.200 more than they originally expected.

a) If the original number of people was x, **write an expression** of how much each was to contribute.

b) **Write down** an expression on how much each contributed after the five pulled out.

c) **Calculate** how many people made the contribution

d) **How** much did each contribute?

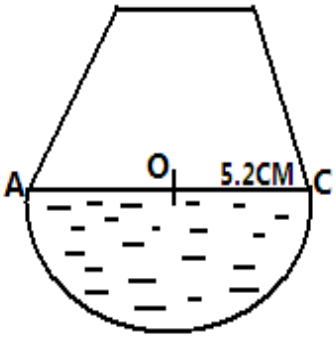
3. (a) In 2001 the total cost of manufacturing an article was Sh.1250 and this was divided between the cost of material, labour and transport in the ratio 8: 14: 3. In 2004 the cost of the

material was doubled, labour cost increased by 30% and transport costs increased by 20%.

**Calculate** the cost of manufacturing the article in 2004.

(b) For the same article in (a) above, the cost of manufacturing in 2005 was sh. 1981 as a result of increase in labour costs only. **Find** the percentage increase in labour cost of 2004.

4.(a) The diagram below shows a cross-section of a bottle. The lower part ABC is a hemisphere of radius 5.2cm and the upper part is a frustum of a cone. The top radius of the frustum is one third of the radius of the hemisphere. The hemispherical part is completely filled with water as shown in the diagram.

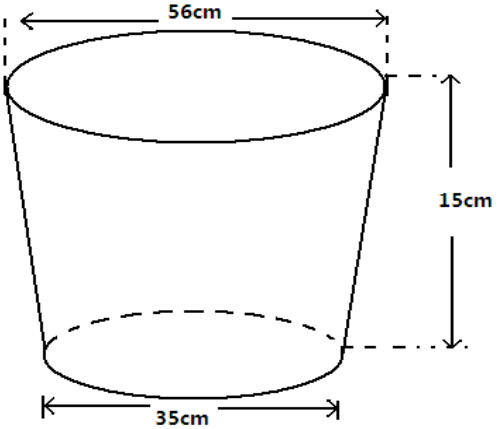



When the container is inverted, the water now completely fills only the frustum part.

e) **Determine** the height of the frustum part

f) **Find** the surface area of the frustum part of the bottle

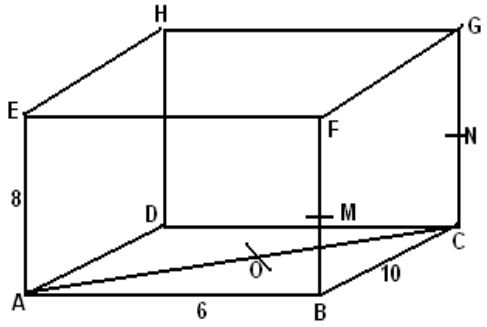
(b) The figure below represents a basin whose top diameter is 56cm bottom diameter is 35cm and depth is15cm




a) **Calculate** the capacity of the basin in litres.

b) **Calculate** the highest of a cylindrical container whose base radius is 31.5cm that can be exactly filled by twice the volume of the water from the basin above

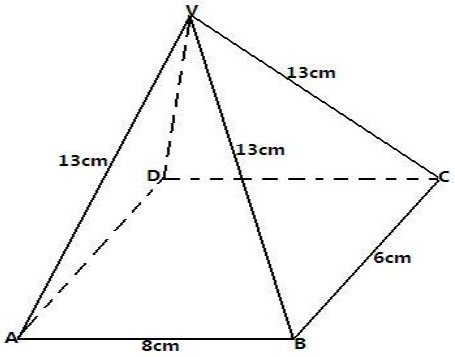
5. (a) In the cuboid below, AE=8cm AB=6cm and BC =10cm. M and N are mid –points of BF and CG. O is the mid- point of AC.




**Calculate:**

(a) the length of EC
(b) the angle between the planes ONM and BCGF
(c) the angle between the lines AF and BE

(b) ABCDV is a rectangular based pyramid with AB =8cm, BC =6cm and VA =VB =VC =VD =13cm.




a) **Draw** the net of the solid
b) A line is drawn from V through the mid points of BC and AD to V. **Calculate** the length of this line.
c) **Determine** the angle the plane VBC makes with plane VAD

6. Two towns P and Q are 550km apart. A bus starts from town Q towards P at 8:45 am at average speed of 80km/h. A car starts from P towards Q at 10:00 am at an average speed of 100km/h, **Calculate**

   a. The distance covered by the bus before the car starts its journey

   b. How far from Q the two vehicles meet

   c. The time the two vehicles met

   d. The time the car arrived at town p.

7. A plot is in the form of a trapezium with sides AB =74m, BC =48m, CD =56m, Angle ABC $=81^0$ and AB parallel to DC.

a) Using a scale 1cm to 10m, **construct** the plan of the plot.
b) On the plan **construct** locus $L_1$ ,of points equidistant from sides AB and AD and locus $L_2$ of points equidistant from sides DC and DA
c) If $L_1$ and $L_2$ meet at M, a TV mast, **locate** M

d) **Shade** the region inside the plot where trees can be planted such that they are at least 25m away from the mast.

8. Three bags A, B and C each has some coloured balls. The probability of picking a black ball from bag A is 3/7 from bag B is 7/9 and from bag C is 1/8. Two balls are picked from the bags and the first bag to be picked from is bag A. If the ball is black then the second ball is from bag B otherwise it would be bag C

a) **Determine** the smallest number of balls in each bag
b) **Represent** the above information on tree diagram
c) **Find** the probability that:
   (i) Two balls picked are black
   (ii) At least a ball picked is black.

9.The table below shows the annual income rates for Public servants in the year 2001.

| INCOME (£ p.a ) | %TAX |
|---|---|
| 1 – 4,800 | 2 |
| 4,801 – 9,600 | 3 |
| 9,601 – 14,400 | 5 |
| 14,401 – 19,200 | 7 |
| 19,201 – 24,000 | 9 |
| 24,001 - OVER | 10 |

Milly's monthly earnings were as follows:

| | |
|---|---|
| Basic salary | £24,000 |
| House allowance | £ 12,000 |
| Medical Allowance | £ 1,800 |

a) Using the tax table above **calculate** B's PAYE if his family relief is £1410 per month

b) If B pays £ 280 for the national health insurance fund, £ 3200 for hire purchase and £ 5,250 for loan repayment. **calculate** his net salary

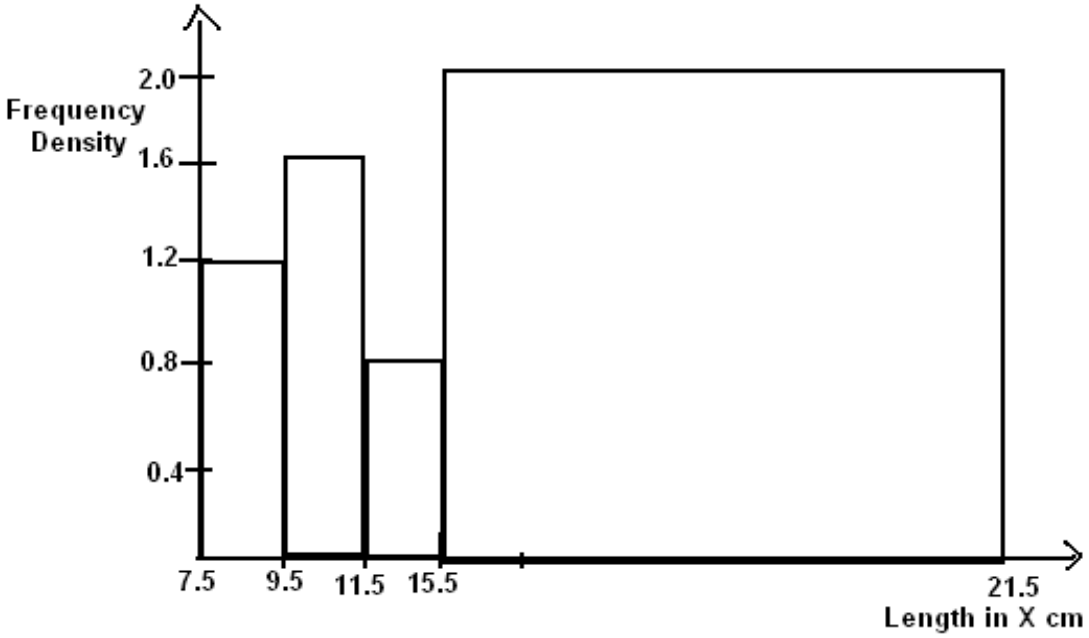
10. The data below shows the ages of some students in a certain collage given to the nearest year.

| Age in years | 17 – 19 | 20 – 22 | 23 – 25 | 26 – 28 | 29 - 31 |
|---|---|---|---|---|---|
| No. of Students | 3 | 7 | 13 | 25 | 12 |

a) On a graph paper **draw** a cumulative frequency curve for the data

b) Use the graph in (a) above to **determine:**
   i. The median and semi –interquartile range
   ii. The $80^{th}$ percentile age
   iii. The percentage of students whose ages lie in the range 24yrs to 30yrs

1 1.The figure below shows a histogram.




Frequency density = $\frac{frequency}{classwidth}$

**Fill in** the table below the missing frequencies:

| Length 10xcm | Frequency |
|---|---|
| $7.5 \le \times \le 9.5$ | 12 |
| $9.5 \le \times \le 11.5$ | |
| $11.5 \le \times \le 15.5$ | |
| $15.5 \le \times \le 21.5$ | |

(a) Using the graph estimate the mode
(b) Calculate the mean length
(c) Calculate the mode and median.

12. A matrix $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ represents the transformation T, triangle ABC where A(1,1) B(5,1) and C(2,4) is transformed by T.

a) i) Find the image $A^1B^1C^1$ of ABC under T.

ii) Draw $A^1B^1C^1$ and ABC

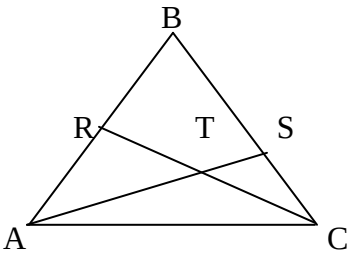
iii) Describe the transformation T

b) Draw $A^2B^2C^2$ image $A^1B^1C^1$ under enlargement centre (0,0) scale factor $-\frac{1}{2}$

c) Find a single matrix that would $A^2B^2C^2$ onto ABC.

d) What is the ratio of the area of ABC to the area of $A^2B^2C^2$.

13. In triangle ABC $\underset{\sim}{AB} = b$ and $\underset{\sim}{AC} = c$. A point R divides AB in the ratio 1:3 and S divides BC in the ratio 5:2. AS and CR meet at T.

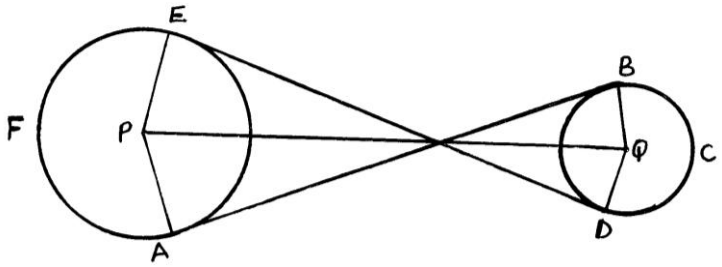



a) Using $\underset{\sim}{b}$ and $\underset{\sim}{c}$ find an expression for
   i) $\underset{\sim}{AS}$  ii) $\underset{\sim}{CR}$

b) If $\underset{\sim}{AT} = h\underset{\sim}{AS}$ and $\underset{\sim}{CT} = k\underset{\sim}{CR}$ where h and k are scalars by expressing AT in two different ways find the values of h and k.

c) Hence find the ratio RT: TC

14. The figure below shows two pulleys whose centres are 30cm apart connected by a belt ABCDEF. The pulley centre P has a radius 13cm and the pulley centre Q has a radius of 4cm.




Calculate

(a) The length AB

(b) The reflex angles EPA and BQD.

(c ) The arc length AFE and BCD.

( d) The total length of the belt.

15. (a) Complete the table below giving your values correct to 2 decimal places. (2mks)

| x | $0^0$ | $15^0$ | $30^0$ | $45^0$ | $60^0$ | $75^0$ | $90^0$ | $105^0$ | $120^0$ |
|---|---|---|---|---|---|---|---|---|---|
| 3 Cos $x^0$ | | | 2.60 | | | | 0 | -0.75 | |
| 4 Sin $(2x-10^0)$ | -0.69 | 1.37 | | | | | | | -3.06 |

(b) Taking 1cm to represent $15^0$ on the x-axis and 2cm to represent 1 unit on the y-axis. Draw the graphs of $y=3\cos x^0$ and $y= 4 \sin (2x - 10^0)$ on the same set of axes .

c) Use your graph to find the values of x for which $3 \cos x - 4 \sin (2x - 10^0) = 0$.

16. A school has to take 384 people for a tour. There are two types of buses available. Type X and type Y. Type X can carry 64 passengers and type Y can carry 48 passengers. They have to use at least 7 buses.

a) Form all linear inequalities which will represent the above information

b) On a graph paper, draw the inequalities and shade the un-wanted region

The charges for hiring the buses are

Type X: sh. 25,000

Type Y: sh 20,000

c) Use your graph to determine the number of buses of each type that should be hired to minimize the cost.

17. A tailoring business makes two types of garments, A and B. Garment A requires 3 metres of material while garment B requires 2 ½ metres of material. The business uses not more than 600 metres of material daily in making both garments. It must not make more than 100 garments of type A and not less than 80 of type B each day.

(i) **Write** down four inequalities from this information.

(ii) **Graph** these inequalities.

(iii) If the business make a profit of Sh. 80 on garment A and a profit of sh. 60 on garment B, **how many** garments of each type must it make in order to maximize its total profit? (Assume that all the garments made are sold in the same day).

18. (a) Given that $x - y = 3$ and $3x + y = 17$. **Find** without solving for x and y the value of

(i) $2xy - x^2 - y^2$

(ii) $6xy + y^2 + 9x^2$

(iii) $3x^2 - 2xy - y^2$

(iv) $\dfrac{3x^2 - 4xy + y^2}{9x^2 - y^2}$

(b) Complete the table below for the function $y = X^3 + 6X^2 + 8X$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| X | -5 | -4 | -3 | -2 | -1 | 0 | 1 |
| $X^3$ | -125 | -64 | | -8 | -1 | 0 | 1 |
| $6X^2$ | | 96 | 54 | | 6 | 0 | 6 |
| 8X | -40 | | -24 | -16 | | 0 | 8 |
| Y | | | 3 | 0 | -3 | 0 | 15 |

(a) **Draw** the graph of the function $y = x^3 + 6x^2 + 8x$ *for* $-5 \leq \times \leq 1$
(Use a scale of 2cm to represent 1 unit on the x axis. 1 cm to represent 2 units on the y-axis)

(b) Hence use your graph to estimate the roots of the equations.

(i) $x^3 + 6x^2 + 8x = 0$

(ii) $x^3 + 5x^2 + 4x = -x^2 - 3x - 1$

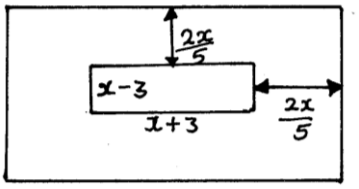
(c) Find the values of x which will satisfy the inequality $x^3 + 6x^2 + 8x > 1$

19. The figure below represents the floor of a dancing hall with a carpeted margin all around of $\frac{2x}{5}$ m wide leaving a dancing space of ($x$-3)m by ($x$+3)m

a) If the total area of the entire room is 315$m^2$, calculate the value of $x$

b) Hence calculate the area of the carpeted margin.

c) If the carpet cost shs. 750 per $m^2$. Calculate the total cost of the sealed margin.




20. a) Given the matrices

$$A\begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix} \quad B\begin{pmatrix} 1 & -2 \\ 3 & 1 \end{pmatrix} \text{ and } \quad C\begin{pmatrix} 2 & 0 \\ -1 & 1 \end{pmatrix}$$

Find a matrix M such that $M=2AB + 3C^2$.

b) a boarding school uses 15 bags of maize, 8 bags of beans, 16 bags of maize flour and 4 bags of rice in the first term. The prices are sh.1000, sh.1,200, sh.1400 and sh.1,400 respectively. In the second term, the school uses 16 bags of maize, 10 bags of beans 18 bags

of maize flour and 5 bags of rice at sh. 1400, sh.2600, sh.1600 and sh.1500 respectively. In the third term the school uses 12 bags of maize 5 bags of beans, 12 bags of maize flour and 3 bags of rice at sh.1800, sh.2200, sh.2,000 and sh.1,500 respectively. Using a matrix method, find the total cost of foodstuff that year.

21. A tourist office looks at 20 East African national parks, to see which of them have elephants (E), lions (L) and rhinoceros (R). They find that

$n(E \cap L^1 \cap R^1) = 2$, $n(L \cap E^1 \cap R^1) = 1$, $n(R \cap L^1 \cap E^1) = 0$ $n(E \cup L \cup R)^1 = 3$
$n(E \cap L \cap R^1) = 8$

5 parks have elephants and rhinoceros and 4 parks have lions and rhinoceros.

a) Draw a venn diagram to show this information.

b) How many parks had all three of these animals?

c) What is the probability that a park chosen at random has at least two of these animals?