P425/1
PURE MATHEMATICS
Paper 1
June/July 2022
3 Hours

MWALIMU EXAMINATIONS BUREAU

UACE RESOURCE MOCK EXAMINATIONS 2022
PURE MATHEMATICS

PAPER 1

3 Hours

INSTRUCTIONS TO CANDIDATES

*Answer **all** the **eight** questions in Section **A** and any **Five** from Section **B.***

*All necessary working **must** be shown clearly.*

Graph paper is provided.

Begin each answer on a fresh page.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 MARKS)

1 Solve the simultaneous equations $xy = 80.$
$\log x - 2\log y = 1$ **(5 marks)**

2 Points A(2,1) and B lie on the curve $y = \frac{(x-1)(x+2)}{2x}$, if the tangent at B is parallel to the tangent at A. determine the coordinates of point B. **(5 marks)**

3 Solve for θ in the interval $0 \leq \theta \leq 180^0$ in the equation
$3\,sin^2\theta + 5\sin\theta\cos\theta - 2cos^2\theta = 0.$ **(5 marks)**

4 Given $y = (2x-1)^3(x+1)^4$, find the values of x for which $\frac{dy}{dx} = 0.$ **(5 marks)**

5 Given $A(2,2)$ *and* $B(8,8)$ are fixed points such that AP = 2PB. Show that the locus of P is a circle and determine its radius. **(5 marks)**

6 Find the equation of the plane through the three points
$(2.-1,4)$, $(3,2,-6)$ and $(4,1,5)$ **(5 marks)**

7 Express $(x^2+x^2)(x-1)^2 = x^2$ in a simplified polar form. **(5 marks)**

8 Show that $\int_1^e \frac{1+\ln x}{x}\,dx = \frac{3}{2}.$ **(5 marks)**

SECTION B (60 MARKS)

9 $A\ (0,1,3)$, $B\ (4,5,-5)$, $C\ (-3,0,-1)$ and $D\ (7,5,4)$ are points in three dimensional planes.

(i) Show that point P divides AB in the ratio 1 : 3 and also divides CD in the ratio 2 : 3.

(5 marks)

(ii) Find the length of AP. **(2 marks)**

(iii) Find the angle between the lines AB and CD. **(5 marks)**

10(a) Prove that $\frac{1+\sin x\cos x}{cos^2 x} = tan^2 x + \tan x + 1$.

Hence find the minimum value of $\frac{1+\sin x\cos x}{cos^2 x}$. **(5 marks)**

(b) Show that

$$4(1+\cos\theta+\cos 2\theta) - 3(\sin\theta + \sin 2\theta) = (1+2\cos\theta)(a\sin\theta + b\ \sin\theta)$$

Where a and b are values to be determined. Hence solve the equation $4(1+\cos\theta+\cos 2\theta) - 3(\sin\theta+\sin 2\theta) = 0$ for $0 \le \theta \le 360^0$ **(7 marks)**

11(a) Given $P(t^2, 3t)$ is a variable point on the parabola$y^2 = 9x$ and point O is the origin of the axes. Find the locus of point M the midpoint of OP. **(3 marks)**

(b) Prove that $y = mx + c$ is a tangent to the parabola $y^2 = 4ax$ if $c = \frac{a}{m}$. **(3 marks)**

Hence find the equations of the tangents from $(1,3)$to the parabola $y^2 = -16x$

(6 marks)

12(a) Expand $\left(x+\frac{1}{x}\right)^5 - \left(x-\frac{1}{x}\right)^5$. Hence evaluate $2.5^5 - 1.5^5$ **(6 marks)**

(b) If α , β are roots of the quadratic equation $x^2 - 4\sqrt{2}\ kx + 2k^4 - 1 = 0$ and $\alpha^2 + \beta^2 = 66$.

(i) Determine the positive value of k. **(3 marks)**

(ii) Determine the value of $\alpha^3 + \beta^3$. **(3 marks)**

13.(a) The first term of a geometric progression is 81 and the fourth is 24. Find:

(i) the common ratio of the progression. **(4 marks)**

(ii) the sum to infinity of the progression. **(2 marks)**

(b) The second term and third terms of this progression are the first and fourth terms respectively of an arithmetic progression. Find the sum of the first ten terms of the arithmetic progression. **(6 marks)**

14(a) Given $y = \frac{x}{\sqrt{1+x^2}}$, prove that $(1+x^2)\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} = 0$. **(6 marks)**

(b) If the equation of a tangent to the curve $y = \frac{1}{3}x^3$ at $\left(1,\frac{1}{3}\right)$is given by $ax + by + c = 0$. Find the equation of the perpendicular to this tangent which is itself a tangent to the curve $y = \frac{4}{x^2}$.

(6 marks)

15 (a)Express $\frac{4-x}{(1+2x)(2+x^2)}$ in the form $\frac{A}{1+2x} + \frac{Bx+C}{2+x^2}$. Hence evaluate $\int_0^1 \frac{4-x}{(1+2x)(2+x^2)}dx$.

(6 marks)

(b) Sketch the curve $y = 4x - x^2$. Hence find area of the region enclosed by the lines x = 1, x = 2, y = 0 and the curve. **(6 marks)**

16(a) Solve the differential equation $\frac{dy}{dx} + \frac{4y}{x} = 6x - 5$ and y =1 when x = 1. **(4 marks)**

(b) The rate at which an object cools is proportional to the temperature difference between the object and the surrounding temperature of $25^0 C$. Warm water for bathing is poured in a basin and cools from $41.1^0\ C$ to $40^0\ C$ in 5 minutes.

(i) Form a differential equation to represent this situation and solve it to determine the temperature of the water after 10 minutes. **(6 marks)**

(ii) Find the time taken for the water to cool to $37^0\ C$. **(2 marks)**

END