SENIOR SIX MATHEMATICS SEMINAR TO BE HELD ON 27TH FEBRUARY 2021 ONLINE SEMINAR THROUGH ZOOM.

APPLIED MATHEMATICS P425/2

Instructions:

As a school, please select several questions and prepare presentations in each. We shall book the questions of choice through the WhatsApp group and on the seminar day.

STATISTICS/PROBABILITY:

1. Box A contains 4 red and 5 blue pens, while box B contains 4 red and 4 blue pens. It is known that box A is three times likely to be selected as box B. A box is selected at random and two pens are drawn from it without replacement.

   (a) Find the probability of picking pens of different colours.

   (b) Construct a probability distribution for the number of blue pens that are drawn. Hence calculate the variance of the number of pens drawn from the box.

2. The following table gives the distribution of the interest paid to 500 shareholders of Moringa Growers' Association at the end of 2000.

Interest x 1000 USh	25 -	30 -	40 -	60 -	80 -	110 -	120- < 130
No. of share holders	17	55	142	153	93	20	20

   i) Draw a histogram to illustrate this data.

   ii) Find the average interest each shareholder receives.

   (iii) Calculate the standard deviation of the distribution above.

   (iv) Using a cumulative frequency curve or otherwise, find the range of the interest obtained by the middle 80% of the shareholders.

(b) A four-man team is to be selected from three women and 4 men,

   (a) What is the probability that (i) the women will form the majority?

   (ii) at least 3 men will be on the committee?

(b) If the group contains a couple that insists on being on the committee together repeat question (a)(i) and (ii) above.

3. In a machine manufacturing company, 85% of the nails made are approximately within the set tolerance limits. If a random sample of 200 nails is taken, find the probability that;
   (i) exactly 33 are outside the tolerance limits.
   (ii) between 21 and 27 nails, inclusive, will be outside the tolerance limits.
4. The continuous random variable X has a probability function,

$$f(x)=\begin{cases} k(x+2); & -1<x<0 \\ 2k; & 0\leq x\leq 1 \\ \frac{k(5-x)}{2}; & 1<x\leq 3 \\ 0, & elswhere \end{cases}$$

   where k is a constant.
   (a) Sketch f(x) and hence find the constant, k
   (b) median
   (c) $P(\frac{1}{2}<x<2)$
5. (a) The weights of ball bearings are normally distributed with mean 25gram and standard deviation 4 grams. If a random sample of 16 ball bearings is taken, find the;
   (i) Probability that the mean of the sample is between 24.12 grams and 26.73 grams.
   (ii) Interquartile range.
   (c) A random sample of 120 girls taken from a normally distributed population of girls school gave a mean age of 16.5 and variance of 18. Determine the 97% confidence interval for the mean age of all the school girls.

MECHANICS:

6. (a) A car traveling at 54 kmh^{-1} is brought to rest with uniform retardation in 5 seconds. Find its retardation in ms^{-2} and distance it travelled in this time

   (b) A cyclist was timed between successive trading centres P; Q and R, each 2 km a part. It took $\frac{5}{3}$ minutes to travel from P to Q and 2.5 minutes from Q to R. Find;

   (i) the acceleration

   (ii) the velocity with which the cyclist passes point P

   (iii) How much further the cyclist will travel before coming to rest if the acceleration remains uniform.

7. A bullet is fired from a point P which is at the top of a hill 50m above the ground. The speed with which the bullet is fired is $140ms^{-1}$ and it hits the ground at a point Q which is at a horizontal distance 200m from the foot of the bill. Find the,

   (i) two possible values of angle of projection

   (ii) two possible times of flight.

   (iii) angle with which the bullet hits the grounds.

8. Two smooth inclined planes meet at right angles, the inclination of one to the horizontal being 30° and the other being 60°. Bodies of masses 2kg and 4kg lie on the respective planes, and the two masses are joined by a light inextensible string passing over a smooth fixed pulley at the intersection of the planes.

   (i) If the masses are released from rest, find the acceleration of the masses and tension of the string.

   (ii) If the surface with 4kg mass experiences a frictional force of 0.25N, find the new acceleration of the masses when the system is set from rest.

9. (a) A pump draws water from a tank and issues it at a speed of 8 ms^{-1} from the end of a pipe of cross-sectional area of $0.01m^2$ situated 10m above the level from which the water is drawn. Find the rate at which the pump is working (take density of water = $1000kgm^{-3}$)

(b) A car of mass 800kg is pulling a trailer of mass 200kg up a hill inclined at an angle $Sin^{-1}\left(\frac{1}{14}\right)$ to the horizontal. When the total force exerted by the engine is 1000N the car and the trailer move up the hill at a steady speed.

(i) Find the total frictional resistance to the motion of the car and trailer during this motion

(ii) If the frictional resistance on the car is 280N, find the tension in the coupling between the car and the trailer.

10. A particle A initially at the point with position vector $2\mathbf{i} - 5\mathbf{j} + \mathbf{k}$ km is moving with a constant velocity of $\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ kmh^{-1}. At the same instant, a particle B at the point $(3,3,2)$is moving with a constant velocity of $3\mathbf{i} - 2\mathbf{k}$ kmh^{-1}. Find the:

(i) relative velocity of particle A to B.

(ii) relative displacement of particle A to particle B at any instant.

(iii) shortest distance between the two particles in their subsequent motion.

11. A body moving with an acceleration $(2e^{4t}\boldsymbol{i} - 3Cost\,\boldsymbol{j} + 4Sin2t\,\boldsymbol{k})$ m/s^2 is initially located at a point whose position vector is $(\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k})m$ and has a velocity $(\boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k})\ m/s$ at time t = 0.

Find the (a) velocity of the body at time t= $\frac{\pi}{2}$s

(b) displacement of the body at any time t =1s.

12. A non-uniform ladder AB whose centre of gravity is 2m from end A is of length 6m and weight, **W**. The ladder is inclined at an angle θ to the vertical with its end B against a rough vertical wall and end A on a rough horizontal ground with which the coefficients of friction at each point of contact is μ. If the ladder is about to slip when a man of weight **5W** ascends two-thirds of the way up the ladder, show that $\boldsymbol{tan\theta = \frac{18\mu}{11-7\mu^2}}$

13. (a) ABCD is a square of side 2m. Forces of magnitudes 3N, 5N, 7N and 2N act a long sides DA, AB, BC and CD respectively. Calculate the:

(i) magnitude of the resultant of the forces and the angle made by the resultant with AD.

(ii) sum of the moments of the forces about A

(iii) distance from A of the point where the line of action of the resultant of the forces cuts DA produced.

(iv) equation of the line of action of the resultant.

14. (a) A smooth bead of mass 0.2 kg is threaded on a smooth circular wire of radius r metres which is held in a vertical plane. If the bead is projected from the lowest point on the circle with speed $\sqrt{3rg}$. Find the;

(a) speed of the bead when it has gone one sixth of the way round the circle.

(b) force exerted on the bead by the wire at this point.

NUMERICAL METHODS:

15. (a) Give X = 12.6955

i) Truncate to 2 significant figures ii) Round to 2 decimal places

b) Given A = 7.684, B = 0.31, Write down the maximum possible errors in A and B. Hence, find; (i) absolute error in the quotient $\frac{A + B}{AB}$

(ii) Interval within which the quotient in b (i) above lies.

c) The quantities **p** and **q** were measured with errors e_1 and $e_2$respectively. Show that the maximum relative error in $\frac{p}{\sqrt{q}}$ is given by $\left|\frac{e_1}{p}\right| + \frac{1}{2}\left|\frac{e_2}{q}\right|$

Hence find the range within which $\frac{2.35}{\sqrt{5.1}}$ lies

16. a) A school canteen started its operation with a capital of 2 million shillings. At the end of a certain term its profits in soft drinks and foodstuff were 0.2 million and 0.45 million respectively. There were possible errors of 6.5% and 8.2% in the sales

respectively. Find the maximum and minimum values of the sales as a percentage of the capital.

b) The charges of sending parcels by a certain, distributing company depends on the weights, 500g, 1kg, 1.5kg, 2kg and 5kg the charges are 750/=, 1000/=, 2000/=. 3500/=, and 5500/= respectively. Estimate;

i) What would the distributor charge for a parcel of weight 2.7kg?

ii) If the sender pays 6200/=, what is the weight of the parcel?

17. Show that the Newton – Raphson formula for solving the transcendental equation

$\sin x - \frac{1}{2} = 0$ is given by $x_{n+1} = \dfrac{x_n \cos x_n - \sin x_n + \frac{1}{2}}{\cos x_n}$

Show also that one of the roots of the equation lies between 0.5 and 2. Hence find the root to two decimal places.

18. (a) Use the trapezium rule with 6 ordinates to find $\int_0^{\frac{\pi}{2}} \dfrac{1}{1+\cos x}\,dx$ correct to three decimal places.

(b) Calculate the percentage error made in the approximation in (a) above.

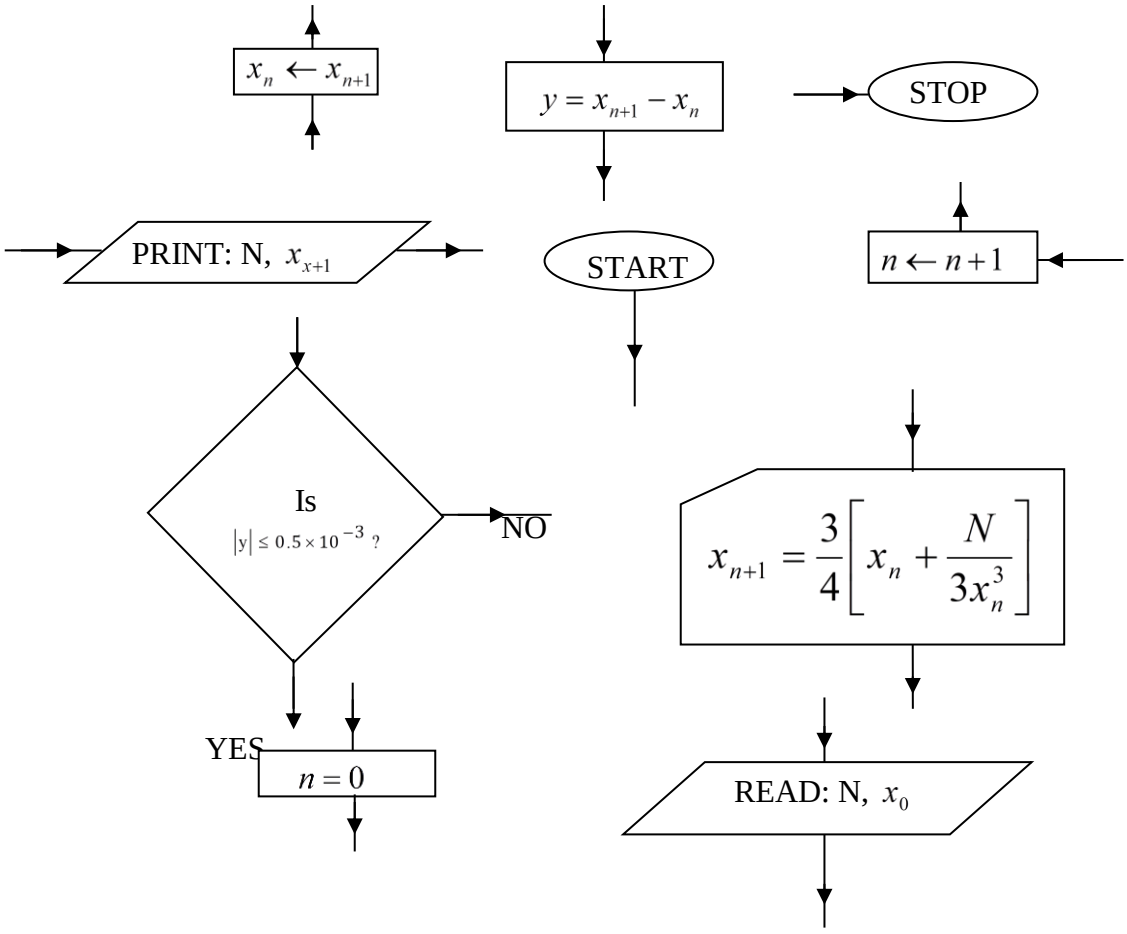
19. Two iterative formulae A and B shown below are used to solve the equation f(x) =0. Only one of these re-arrangements provides a formula which converges to the root.

A	B
$x_{n+1} = \frac{1}{3}(x_n^3 + 1)$	$x_{n+1} = \dfrac{3x_n - 1}{x_n^2}$

(i) Determine the equation whose root is being sought.

(ii) Starting with $x_0 = 0.2$, use the most suitable formula to find the root of the equation f(x) =0 correct to 2 decimal places.

20. Given below are elements of a flow chart not rearranged in order



(i) Re-arrange the elements to form a logical flow chat. State the purpose of the flow chart.

(ii) Perform a dry run of your flow chart, taking N= 187.42 and $x_0 = 3.0$. Give your answer correct to three decimal places.

Format for the presentations:

(a) All schools will receive the same questions and will be required to select questions and book with the organisers their preferred questions for discussion through the WhatsApp platforms.

(b) Each presenter selected to present at the seminar will prepare their presentation and type it out if possible or write it down in a good handwriting with a black pen. These presenters should be supported by their subject teachers where possible. These final presentations will be sent to the organisers to be uploaded on the HeLP site before the seminar day- www.help.sc.ug .

(c) On the seminar day within the schools:

i. Each school will designate a presentation room that will accommodate the team making the presentation and answering questions from the audience. This room will have all the camera work with a fine and long blackboard or white board and a support subject teacher and ICT back stopper.

ii. The school should also arrange for 2-3 other rooms where the rest of the students will sit while observing the health SOPs. Each room should have a projector and laptop connected to the internet with a solid sound system. The school could use a big hall and arrange 3 screens all connected to laptops.

iii. All the teachers of mathematics in the school should be encouraged to attend the seminar to support the students in following keenly the presentations.

iv. On the seminar day, the expert presenter will make a presentation(15min) and answer the questions raised from the audience(10min)

v. The teachers will also make any additional comments(5min)

vi. A time will be provided between presentations to receive comments from UCC, RENU and other partners.

The Holistic eLearning Platform (HeLP) is inviting you to a scheduled Zoom meeting.
Topic: A-LEVEL MATHEMATICS VIRTUAL SEMINAR
Time: Feb 27, 2021 08:30 AM Nairobi

Join Zoom Meeting
https://us02web.zoom.us/j/81410189011?pwd=Rit6YzJCNzA3RzdMOFMrNnJyTUdyZz09

Meeting ID: 814 1018 9011
Passcode: 940152