Name..**Stream**.........**Teacher**..................

DEPARTMENT OF MATHEMATICS

S.6 PURE MATHEMATICS−2020

COVID−19 WEEK 5

PAPER 1

3 HOURS

- *Answer **all** the **eight** questions in section **A** and any **five** from section **B**.*
- *Any additional question(s) answered will **not** be marked.*

SECTION A: (40 MARKS)

1. If α and β are roots of the equation $x^2 - 4x + 2 = 0$, find the quadratic equation whose roots are $\dfrac{\alpha^3 - 1}{\alpha}$ and $\dfrac{\beta^3 - 1}{\beta}$. (*05 marks*)

2. Find the modulus and argument of $\dfrac{(5+i)^4}{(2+3i)^4}$. (*05 marks*)

3. Solve, for the ratio $x : y$, the equation $6x^2 - xy - 12y^2 = 0$. (*05 marks*)

4. Find $\displaystyle\int \frac{x+1}{3+2x^2}dx$. (*05 marks*)

5. Find the equation of a plane through the point $(1, 2, 3)$ and perpendicular to the vector $4\mathbf{i} + 5\mathbf{j} + \mathbf{k}$. (*05 marks*)

6. Differentiate $\dfrac{e^{x^2}\sqrt{\sin x}}{(2x+1)^3}$. (*05 marks*)

7. A point P is twice as far from the line $x + y = 5$ as from the point $(3, 0)$. Find the locus of P. (*05 marks*)

8. Solve $xy^2 + x^2y\dfrac{dy}{dx} = \sec^2 2x$. (*05 marks*)

SECTION B: (60 MARKS)

9. Show that $\displaystyle\int_0^{\frac{1}{2}} \frac{1}{1-x^4}dx = \frac{1}{4}\ln 3 + \frac{1}{2}\arctan\left(\frac{1}{2}\right)$. *(12 marks)*

10. (a) Prove that the sum of the first n terms of an arithmetic progression with first tern a and common difference d is given by

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right).$$

(b) The first term of an A.P is -12 and the last term is 40. If the sum of the progression is 196, find the number of terms and the common difference. *(12 marks)*

11. Investigate the staionary values of $\dfrac{x^3}{1+x^2}$ and sketch the graph of $y = \dfrac{x^3}{1+x^2}$. *(12 marks)*

12. The locus of P is such that the distance OP is half the distance PR, where O is the origin and R is the point $(-3, 6)$.

(i) Show that the locus of P describes a circle in the $x - y$ plane.

(ii) Determine the radius and centre of the circle.

(iii) Where does P cut the line $x = 3$? *(12 marks)*

13. (a) Solve $\sin 3x + \dfrac{1}{2} = 2\cos^2 x$ for $0 \leq x \leq 360^0$.

(b) Prove that in any given triangle ABC $\tan\left(\dfrac{A-B}{2}\right) = \dfrac{a-b}{a+b}\cot\left(\dfrac{C}{2}\right)$. *(12 marks)*

14. (a) Without using tables or calculators, simplify

$$\frac{\left(\cos\frac{\pi}{17} + i\sin\frac{\pi}{17}\right)^8}{\left(\cos\frac{\pi}{17} - i\sin\frac{\pi}{17}\right)^9}$$

(b) Given that x and y are real values, find the values of x and y which satisfy the equation:

$$\frac{2y+4i}{2x+y} - \frac{y}{x-i} = 0.$$

(06 marks)

15. (a) Solve the differential equation $\tan x \dfrac{dy}{dx} - y = \sin^2 x$.

(b) An inverted cone with a vertical angle of 60^0 is collecting water leaking from a tap at a rate of $0.2 \text{ cm}^3\text{s}^{-1}$. If the height of water collected in the cone is 10 cm, find the rate at which the surface area of water is increasing. *(12 marks)*

16. (a) Find, in Cartesian form, the equation of the line that passes through the points $A(1, 2, 5)$, $B(1, 0, 4)$ and $C(5, 2, 1)$

(b) Find the naagle between the line $\dfrac{x+4}{8} = \dfrac{y-2}{2} = \dfrac{z+1}{-4}$ and the plane $4x + 3y - 3z + 1 = 0$. *(12 marks)*

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