

P425/2
APPLIED
MATHEMATICS
Paper 2
July / August, 2019
3 hours

INTERNAL MOCK EXAMINATIONS

Uganda Advanced Certificate of Education

APPLIED MATHEMATICS

PAPER 2

3 hours

INSTRUCTIONS TO CANDIDATES

*Attempt **all** the questions in section A and only **five** questions from section B*

All the working must be shown clearly.

*Extra numbers done will **NOT** be marked.*

Mathematical tables with a list of formulae and squared paper are provided.

Silent, Non - programmable calculators may be used.

Where necessary, take g to be 9.8ms^{-2}

SECTION A (40 MARKS)

1. A and B are mutually exclusive events such that $P(A) = 0.5$ and $P(B) = 0.4$. Find:

(i) $P(A' \cap B')$	(ii) $P(A' \cup B)$ (5 Marks)
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2. A particle is projected with a velocity of 28 ms^{-1} at an angle of elevation 30° . Find how long the particle is at least 9 metres above the point of projection. (5 Marks)
3. Using 2010 as the base year, the price indices of a commodity in 2012 and 2015 are 118 and 125. Calculate the price index of the commodity in 2010 and 2015 when 2012 is taken as the base year. (5 Marks)
4. A particle is initially moving with a velocity $-2\mathbf{i} + 5\mathbf{j}$. Its acceleration at a time t is given by $\mathbf{a} = -4t\mathbf{i} + 6\mathbf{j} + 2t\mathbf{k} \text{ ms}^{-2}$. Find the speed of the particle when $t = 5 \text{ s}$. (5 Marks)
5. The table below shows an extract for $\cos x^\circ$.

$x = 30^\circ$	$6'$	$12'$	$18'$
$\cos x^\circ$	0.8652	0.8643	0.8634

Use linear interpolation or extrapolation to find:

(i) $\cos 30^\circ 25'$	(ii) $\cos^{-1} 0.8647$. (5 Marks)
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6. The weights of packets of wheat flour in a certain factory are normally distributed with mean 3.0 kg and variance 2.89 kg^2 . Find the probability that a random sample of 150 packets has sample mean between 2.8 kg and 3.3 kg. (5 Marks)
7. Three forces of magnitude 5 N, 10 N and 8 N act along the sides AB, BC and AC respectively of an equilateral triangle ABC. Find the resultant force and show that the line of action of the resultant force intersects AB at a distance of $\frac{5}{9}a$ from A. (5 Marks)

8. Construct a flow chart that can be used to find the mean of the cubes of the first n counting numbers. Perform a dry run for $n = 10$. (5 Marks)

SECTION B

Answer **only** five questions in this section.

9. (a) A uniform rod AB of mass 15 kg hangs vertically with end A freely hinged to a fixed point. The rod is pulled aside by a horizontal force F applied at B until it rests in equilibrium making an angle of 25° with the downward vertical. Determine the value of F . (5 Marks)
- (b) A ladder AB rests with its end A on a rough horizontal floor and the other end B against a rough vertical wall, the coefficients of friction being μ_1 and μ_2 at the floor and on the wall respectively. If the centre of gravity of the ladder is at distances a and b from the ends A and B, show that when the ladder is in limiting equilibrium, the inclination of the ladder with the horizontal is $\tan^{-1}\left(\frac{a - b\mu_1\mu_2}{(a + b)\mu_1}\right)$. (7 Marks)
10. (a) The weights of 400 books in a bookshelf are normally distributed with mean 50 g and standard deviation δ g. Given that 99.4% of the books weigh above 45g, find how many books weigh less than 52.48 g. (5 Marks)
- (b) A random sample of 100 taxis inspected on a road on a particular day gave the ages(in years), X , summarized below: $\sum X = 560$, $\sum X^2 = 3,286$. Determine the:
- (i) unbiased estimate for the variance of all the taxis on the road.
- (ii) 91.86% confidence limits for the mean age of all the taxis that operate on the road.

(7 Marks)

11. Particle A travelling with a uniform velocity $3\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$, passes through a point with position vector $5\mathbf{i} - 9\mathbf{j} + 2\mathbf{k}$. At the same instant, particle B travelling with a velocity $4\mathbf{i} + \mathbf{j} - 8\mathbf{k}$ passes through a point with position vector $3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$.

Find the:

- (a) position vectors of A and B after a time, t ,
- (b) minimum distance between A and B in the subsequent motion.

(12 Marks)

12. (a) Use the trapezium rule with 7 ordinates to estimate $\int_1^4 \frac{1}{x(1 + \ln x)} dx$ correct to three decimal places.

- (b) Calculate the exact value of $\int_1^4 \frac{1}{x(1 + \ln x)} dx$.

- (c) Determine the percentage error made in your estimation in (a). State how you can improve your accuracy.

(12 Marks)

13. (a) Two events X and Y are such that $P(X) = \frac{1}{2}$, $P(X/Y) = \frac{2}{3}$ and

$$P(X/Y) = \frac{3}{7}. \text{ Find } P(X \text{ or } Y). \quad (7 \text{ Marks})$$

- (b) A hot vending machine supplies Coffee, Tea and Ginger, the demands for which are in the ratio 4: 3 : 2. The probability that the machine is unable to supply Coffee on demand is 0.05, 0.04 and 0.02 respectively. Given that the machine is unable to supply a particular customer's choice, find the probability that the customer needed coffee. (5 Marks)

14. (a) Show that the equation $e^x + x = 10$ has a root between 2 and 3.
- (b) Use Newton Raphson's method to find the root to the equation in (a) above, correct to three decimal places.

(12 Marks)

15. A continuous random variable X has a probability distribution function

$$\text{defined by } f(x) = \begin{cases} k(3-x); & 1 < x < 2 \\ k & ; \quad 2 < x < 3 \\ k(x-2); & 3 < x < 4 \\ 0 & ; \quad \text{otherwise} \end{cases}, \text{ where } k \text{ is a constant.}$$

- (a) Sketch the graph of $f(x)$ and find the value of k .
- (b) Determine the cumulative distribution function $F(x)$.
- (c) Calculate $P(2.4 < X \leq 3.5)$.

(12 Marks)

16. (a) A particle of mass of 5 kg is placed on an inclined plane of angle 30° to the horizontal. The coefficient of friction between the particle and the plane is 0.2. What angle applied at an angle of 15° to the plane would prevent the particle from sliding down the plane?

(5 Marks)

- (b) A train of mass 250 Mg is traveling up a slope of 1 in 140 at a constant speed of 48 kmh^{-1} .
- (i) Taking the frictional resistance to be $\frac{1}{160}$ of the weight of the train, calculate the power which is being exerted.
- (ii) Find also the maximum speed in km per hour that 450 kW could maintain on the level ground, if the frictional resistance were then increased to $\frac{1}{150}$ of the weight of the train.

(7 Marks)

END