HOLIDAY WORK FOR MATHEMATICS

S.5 GROUP WORK 2

1. If $y = \sec x$, prove that $y\dfrac{d^2y}{dx^2} = \left(\dfrac{dy}{dx}\right)^2 + y^4$.

2. Differentiate with respect x and simplify your answer $\ln\sqrt[3]{\dfrac{2+x^3}{2-x^3}}$.

3. Given that $y = \ln(x^2 + x + 2)$, show that $(x^2 + x + 2)\dfrac{d^3y}{dx^3} + 2(2x+1)\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} = 0$.

4. Find the turning points to the function $y = \tan x - 6\operatorname{cosec} x$ and distinguish between them.

5. Determine the turning points to the curve $y = e^{-x}\sin x$.

6. Simplify $\log_e\sqrt{\dfrac{(x+1)e^{-2x}}{1-x}}$ and show that its derivative is $\dfrac{x^2}{1-x^2}$.

7. If $y = \ln(x^2 - 5)$, show that $\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^2 = 2e^{-y}$.

8. If $y = \sin 2x \ln(\tan x)$, show that $\dfrac{d^2y}{dx^2} + 4y = 4\cot 2x$.

9. Given that $\tan y = \log_e x^2$, show that $x\dfrac{dy}{dx} = 2\cos^2 y$. Hence show that
   $x^2\dfrac{d^2y}{dx^2} + 2(1 + 2\sin 2y)\cos^2 y = 0$.

10. If $\log_e(x^2 + y^2) = \tan^{-1}\dfrac{y}{x}$, prove that $\dfrac{dy}{dx} = \dfrac{y+2x}{x-2y}$.

11. Differentiate $\cos^{-1}\left(\dfrac{3+5\sin x}{5+3\sin x}\right)$ giving your answer in its simplest form.

12. If $y = (\sec x + \tan x)^2$, show that $\cos x\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} = 2y\tan x$.

13. Show that $\dfrac{d}{dx}\left[\tan^{-1}(\sec x + \tan x)\right] = \dfrac{1}{2}$.

14. Use the substitution $x^2 = \tan\theta$ to find (i) $\int \frac{1}{(1+x^2)^2}\,dx$ (ii) $\int \frac{x}{1+x^4}\,dx$

15. Express the following in partial fractions and find the integral in each case.

(a) $\frac{2x}{9-x^2}$ (b) $\frac{2x^2}{(2-x)(4+x^2)}$ (c) $\frac{x^2-8x+5}{(1+2x)(9+x^2)}$

(d) $\frac{11x+12}{(2x+3)(x+2)(x-3)}$ (e) $\int_3^5 \frac{x^3-3}{(x-2)(x^2+1)}\,dx$

16. If $y = \tan\left(2\tan^{-1}\frac{x}{2}\right)$, show that $\frac{dy}{dx} = \frac{4(1+y^2)}{4+x^2}$

17. Given that $y = \sin\theta$ and $x = 1+\cos 2\theta$, show that $\frac{d^2y}{dx^2} = 4\left(\frac{dy}{dx}\right)^3$

18. (a) The points P, Q and R have position vectors $2\mathbf{a} - 5\mathbf{b}$, $5\mathbf{a} - \mathbf{b}$ and $11\mathbf{a} + 7\mathbf{b}$ respectively. Show that P, Q and R are collinear and state the ratio $\mathbf{PQ} : \mathbf{QR}$.

(b) The points A and B have position vectors $4\mathbf{i} + 3\mathbf{j}$ and $\mathbf{i} + t\mathbf{j}$. Determine the values of t such that the angle $A\hat{O}B = \cos^{-1}\frac{2}{\sqrt{5}}$, where O is the origin.

19. Prove by induction that $\frac{1}{3\times 5} + \frac{1}{5\times 7} + \ldots + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$.

20. In a triangle OAB, $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OB} = \mathbf{b}$. Given that E divides OA in the ratio $6:1$, D divides AB in the ratio $1:2$ and point C is on OB produced such that $\mathbf{OC} : \mathbf{OB} = 3:2$, find the ratio $\mathbf{ED} : \mathbf{DC}$.

21. (a) Find the area enclosed by the curve $y = x(3-x)$ and the line $y = 2$.

(b) Determine the volume generated when the area enclosed by the curve $y = x^2 + 4$ and the line $y = 5$ is rotated about the x – axis through 360^0.

22. (a) Use calculus to estimate $\sqrt[4]{63}$ correct to three significant figures.

(b) Use Maclaurin's theorem to expand $y = \ln(x^2+2x+3)$ as far as the term in x^3.

Hence approximate $\ln 3.21$ correct to four decimal places.