**SPECIAL PRACTICE PAPER FOR PURE MATHEMATICS**
**SECTION A: (40 MARKS)**
*Answer **all** questions in this Section.*

1. Given that $\alpha$ and $\beta$ are the roots of the equation $5x^2 - 3x + 2 = 0$, find the equation whose roots are $\frac{2}{\alpha - 2}$ and $\frac{2}{\beta - 2}$. ***(05 marks)***

2. Find the coordinates of the point of intersection of the lines with vector equations $\mathbf{r} = 5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k} + \gamma(-3\mathbf{i} + \mathbf{j} - \mathbf{k})$ and $\mathbf{r} = -2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k} + t(4\mathbf{i} - \mathbf{j} + 2\mathbf{k})$. ***(05 marks)***

3. Solve the equation $2\sin\theta = \sqrt{3}\tan\theta$ for $0^0 \leq \theta \leq 360^0$. ***(05 marks)***

4. Differentiate the following with respect to $x$

   (i) $e^{\tan^2 x}$ (ii) $\frac{x}{\sqrt{x^2+1}}$. ***(05 marks)***

5. Determine the distance between the points of intersection of the curve $5x^2 + 6xy - 8y^2 = 0$ and the line $3x - y = 7$. ***(05 marks)***

6. Find the volume of the solid generated when the area bounded between the curve $y = x\sin x$, the x- axis from $x = 0$ to $x = \frac{\pi}{4}$ is rotated through one complete revolution about the x – axis. ***(05 marks)***

7. Solve the equation $2\times3^{2x+3} - 7\times3^{x+1} - 68 = 0$. ***(05 marks)***

8. A curve is defined parametrically by $x = 3t^3$, $y = t - \frac{1}{t^2}$. Find the equation of the normal to the curve at the point $(-3, -2)$. ***(05 marks)***

## SECTION B: (60 MARKS)

Answer any ***five*** questions from this section

9. Use De Moivre's theorem to simplify

   (i) $\dfrac{\cos\theta + i\sin\theta}{i + \sin 4\theta - i\cos 4\theta}$. ***(05 marks)***

   (ii) $\sqrt{\dfrac{i}{3 - 4i}}$. ***(07 marks)***

*Turn Over*

10. (a) Show that $\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B$. ***(04 marks)***

    (b) (i) Express $8\sin x - 6\cos x$ in the form $R\sin(x - \alpha)$, where $\alpha$ is an acute angle. ***(04 marks)***

    (ii) Hence find the greatest and least values of

    $$\frac{1}{8\sin x - 6\cos x - 7}.$$ ***(04 marks)***

11. (a) It can be proved by induction that

    $$1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{1}{4}n^2(n+1)^2.$$

    Show that $\sum_{r=1}^{n}(r+1)^3 = \frac{1}{4}n(n+3)(n^2 + 3n + 4)$. ***(06 marks)***

    (b) A man invests £ 2,000 at the beginning of each year and receives compound interest at 5% per annum. By forming a series, show that the total amount of accumulated capital and interest at the end of n years is £ $42000(1.05^n - 1)$. Determine also the least number of years that the man would need to continue his investments in order to accumulate not less than £ 100,000.

    ***(06 marks)***

12. P is the point $(ap^2, 2ap)$ and Q is the point $(aq^2, 2aq)$ on the parabola $y^2 = 4ax$. The tangents at P and Q intersect at R.

(a) Find

(i) the equation of the chord joining the points P and Q and hence, deduce the equation of the tangents at P and Q,

(ii) the co-ordinates of R. ***(07 marks)***

(b) Obtain the perpendicular distance of the point R from the straight line PQ and deduce that the area of the triangle PQR is $\frac{1}{2}a^2(p-q)^3$.

***(05 marks)***

13. (a) Using calculus of small changes, find $\sqrt{34}$ correct to **two** decimal places.

***(05 marks)***

(b) By Maclaurin's theorem, expand $\ln\left(\frac{2}{\sqrt{1-4x}}\right)$ as far as the term in $x^3$.

*Turn Over*

***(07 marks)***

14. (a) Integrate $\frac{x}{\sqrt{x^2-4}}$ with respect to $x$. ***(03 marks)***

(b) Find $\int \frac{dt}{1-\cos 4t}$. ***(04 marks)***

(c) Using the substitution $x = 2\cos\theta$, deduce that

$$\int_1^2 \frac{1}{x^2\sqrt{4-x^2}}\,dx = \frac{\sqrt{3}}{4}.$$

***(05 marks)***

15. (a) Find the Cartesian equation of the plane containing the points $A(0, 3, -4)$, $B(7, 4, -1)$ and $C(2, -1, 2)$. ***(06 marks)***

(b) If T is the foot of the perpendicular from $(5, -9, 7)$ to the plane in (a), determine the coordinates of T. ***(06 marks)***

16. (a) Solve the differential equation $y^2 \cos^2 x = \tan x \frac{dy}{dx}$ ***(05 marks)***

(b) An epidemic is spreading through a community at a rate which is proportional to the product of the number of people who have contracted it and that of those who have not contracted it. Given that $x$ is the proportion of people who have contracted the epidemic at time, t,

(i) Form a differential equation relating, $x$ ,t and a constant *k*.

(ii) Show that, if initially a proportion $P_0$ of the people had contracted the epidemic, them $x = \frac{P_0}{P_0 + (1 - P_0)e^{-kt}}$.

***(07 marks)***

***END***