CHAPTER 9: SIMPLE HARMONIC MOTION (S.H.M)

Definition

This is the periodic motion of a body whose acceleration is directly proportional to the displacement from a fixed point (equilibrium position) and is directed towards the fixed point.

$$a \, \alpha - x$$

$$a = -\omega^2 x$$

The negative signs means the acceleration and the displacement are always in opposite direction.

9.1.0: Characteristics of SHM

(1) It's a periodic motion (to and fro motion)

(2) Mechanical energy is always conserved

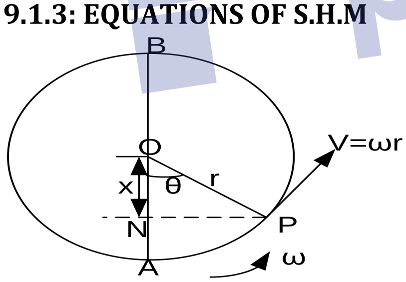
(3) The acceleration is directed towards a fixed point

(4) Acceleration is directly proportional to its displacement

9.1.1: PRACTICAL EXAMPLES OF S.H.M

- Pendulum clocks
- Pistons in a petrol engine
- Strings in music instruments
- Motor vehicle suspension springs
- Balance wheels of a watch

9.1.3: EQUATIONS OF S.H.M



Consider a particle, p of mass, m, moving round a circle of radius ,r, with uniform angular velocity, ω, . Let the particle be at point p, t seconds after starting motion from A such that the angle subtended at the centre is ,θ

a) Displacement x

This is the distance of N from O measured away from O

The displacement is obtained from triangle ONP

$$Cos\theta = \frac{ON}{OP}$$

$$Cos\theta = \frac{x}{r}$$

$x = rCos\theta$ But $\theta = \omega t$

$\boxed{x = rCos\omega t}$-- (1)

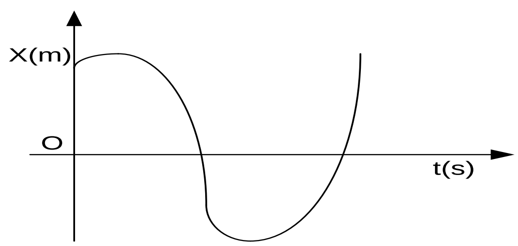
Note:

When the displacement x is maximum i.e. when N is at A or at B, then this displacement is known as **amplitude**.

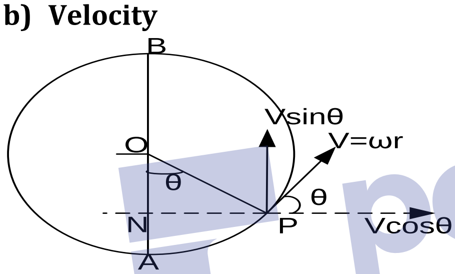
Definition

Amplitude is the maximum displacement of a body (a particle) from equilibrium position.

A GRAPH OF DISPLACEMENTAGAINST TIME



b) Velocity



Velocity of N as a result of the velocity P moving round circle. This is equal to the vertical component of velocity of p

$$v_N = -vsin\theta$$

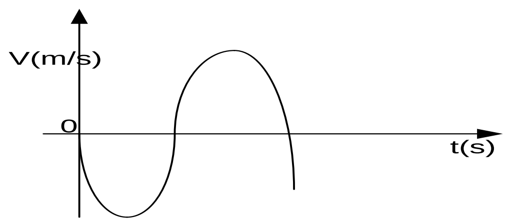
But $\theta = \omega t$ and $v = \omega r$

$\boxed{v_N = -\omega r sin\omega t}$ --------------------------------- (2)

Note:

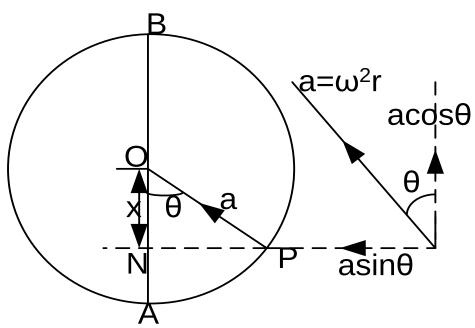
The velocity of N is negative because as p moves from A to B, N moves upwards and when it moves from B to A, N changes direction and moves downwards.

GRAPH OF VELOCITY AGAINST TIME



c) Acceleration $\ddot{x}$ or a

The acceleration of N is as a result of the acceleration of p. This is equal to the vertical component



$a_N = acos\theta$

but $a = \omega^2 r$ and $\theta = \omega t$

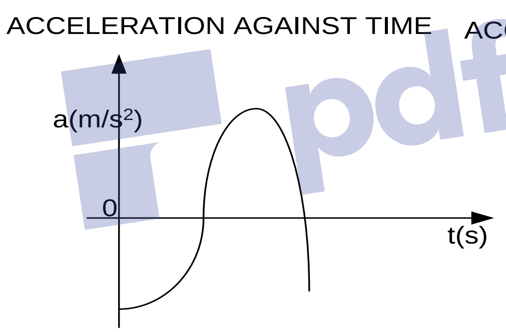
$a_N = \omega^2 rcos\omega t$

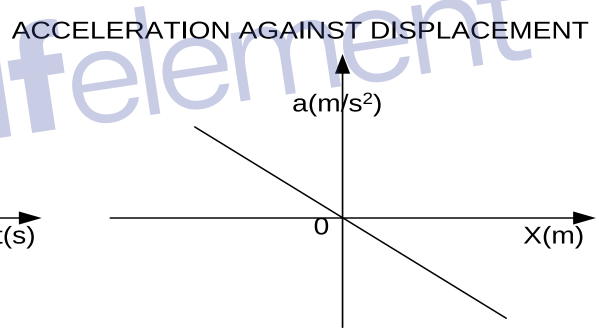
but fom equation 1 $x = rcos\omega t$

$a_N = \omega^2 x$

$\boxed{a_N = -\omega^2 x}$----------------------------------- (3)

$a_{max} = -\omega^2 r$





d) Period T

This is the time taken for one complete oscillation. .i.e. N moving from A to B and back to A.

$$T = \frac{distance}{speed}$$

$T = \frac{2\pi}{v}$ but $v = r\omega$

$T = \frac{2\,\pi\, r}{\omega\, r}$

$T = \frac{2\,\pi}{\omega}$--------------------------- (4)

e) Frequency f

This is the number of complete oscillation made in one second

$f = \frac{1}{T}$

$f = \frac{\boldsymbol{\omega}}{\boldsymbol{2\pi}}$

f) Velocity in terms of displacement

Velocity of a body executing S.H.M can be expressed as a function of displacement x. this is obtained from the acceleration

a= $-\,\omega^2 x$

a= $\frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt}$

but $\frac{dx}{dt} = v$

a = $v \cdot \frac{dv}{dx}$

$v \cdot \frac{dv}{dx}$ = - $\omega^2 x$

vdv = - $\omega^2 xdx$

integrating both sides

$\int v\, dv$ = - $\omega^2 \int x\, dx$

$\frac{v^2}{2} = -\frac{\omega^2 x^2}{2}$ + C ....[1]

Where C is a constant of integration

When t = 0 v=0 and

$x = r$(amplitude)

$\frac{0^2}{2} = -\frac{\omega^2 r^2}{2}$ + C

C = $\frac{\omega^2 r^2}{2}$

Put into [1]

$\frac{v^2}{2} = -\frac{\omega^2 x^2}{2} + \frac{\omega^2 r^2}{2}$

$v^2 = \omega^2 r^2 - \omega^2 x^2$

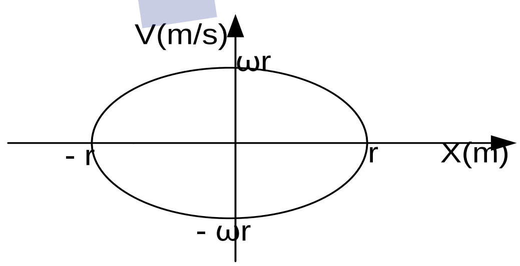
$\boxed{v^2 = \omega^2 (r^2 - x^2)}$

Velocity is maximum when x =0

$v^2 = \omega^2 r^2$

$\boxed{\boldsymbol{v_{max} = \omega r}}$

AGRAPH OF VELOCITY AGAINST DISPLACEMENT



From $\boldsymbol{v^2} = \omega^2 r^2 - \omega^2 x^2$

$\boldsymbol{v^2} + \omega^2 x^2 = \omega^2 r^2$

$\frac{\boldsymbol{v^2}}{\omega^2 r^2} + \frac{x^2}{r^2} = \boldsymbol{1}$

This an ellipse

Example

1. A particles moves in a straight line with S.H.M. Find the time of one complete oscillation when
   i) The acceleration at a distance of 1.2m is 2.4ms^{-2}
   ii) The acceleration at a distance of 20cm is 3.2ms^{-2}

Solution

i) From $a = -\omega^2 x$ Negative is ignored

$2.4 = \omega^2 (1.2)$

$\omega^2 = \frac{2.4}{1.2}$

ω =1.4rads^{-1}

But $T = \frac{2\pi}{\omega}$

$T = \frac{2x\frac{22}{7}}{1.4} = 4.46s$

ii) $a = -\omega^2 x$

$3.2 = \omega^2(0.2)$

ω =4rads^{-1}

$T = \frac{2\pi}{\omega} = \frac{2x\frac{22}{7}}{4}$ =1.57second

2. A Particle moving with S.H.M has velocities of 4ms^{-1} and 3ms^{-1} at distances of 3m and 4m respectively from equilibrium position. Find
   i) amplitude ,
   ii) period ,
   iii) frequency
   iv) velocity of the particle as it passes through equilibrium position

Solution

i) $v = 4ms^{-1}, x = 3m$ and

Using $v^2 = \omega^2(r^2 - x^2)$

$4^2 = \omega^2(r^2 - 3^2)$

$16 = \omega^2(r^2 - 9)$------- (1)

Also $v = 3ms^{-1}, x = 4m$

$3^2 = \omega^2(r^2 - 4^2)$

$9 = \omega^2(r^2 - 16)$------- (2)

Equation 1 divide by 2

$\frac{16}{9} = \frac{\omega^2(r^2 - 9)}{\omega^2(r^2 - 16)}$

$16(r^2 - 16) = 9(r^2 - 9)$

$r^2 = 25$

r=5m

Amplitude =5m

ii) period put r=5m into one of the equations

$4^2 = \omega^2(r^2 - 3^2)$

$16 = \omega^2(5^2 - 9)$

$\omega^2 = 1$

ω =1

But $T = \frac{2\pi}{T}$

T=$\frac{2x\frac{22}{7}}{1}$

T=6.28 seconds

iii) frequency = $\frac{1}{T}$

f = $\frac{1}{6.28}$ =0.16Hz

iv) velocity as it passes equilibrium position at equilibrium x=0

$v^2 = \omega^2(r^2 - x^2)$

$v^2 = 1^2(5^2 - 0^2)$

ω^2=25

$v = 5m/s$

3. A body of mass 200g s executing S.H.M with amplitude of 20mm. The maximum force which acts upon it is 0.064N. calculate
   a) its maximum velocity
   b) its period of oscillation

Solution

F= 0.064N

Mass m=200g = 0.2kg

Amplitude r =20mm = 0.02m

a) $v_{max} = \omega r$

But $F = ma_{max}$

0.064=0.2a_{max}

$a_{max} = 0.32m/s^2$

$a_{max} = -\omega^2 r$

0.32= $\omega^2 x0.02$

ω^2=16

ω =4rads^{-1}

$v_{max} = \omega r = 4x0.02$

v_{max} = 0.08ms^{-1}

b) T= $\frac{2\pi}{\omega}$

T= $\frac{2\pi}{4} = \frac{2x22}{4x7}$

T=1.57 seconds

4. A body of mass 0.30kg executes S.H.M with a period of 2.5s and amplitude of 4x10^{-2}m. determine
   i) Maximum velocity of the body
   ii) The maximum acceleration of the body

Solution

M=0.3kg, T=2.5s, r= $4x10^{-2}$m

vii) $v_{max} = \omega r$

$\omega = \frac{2\pi}{T}$

$$v_{max} = \frac{2\pi}{T} r$$

$$v_{max} = \frac{2x\frac{22}{7}x4x10^{-2}}{2.5}$$

$v_{max} = 0.101m/s$

viii) $a_{max} = \omega^2 r$

$$a_{max} = \left(\frac{2\pi}{T}\right)^2 r$$

$$a_{max} = \left(\frac{2x\frac{22}{7}}{2.5}\right)^2 x\ 4x10^{-2}$$

$$a_{max} = 0.25ms^{-2}$$

5. A particle moves with S.H.M in a straight line with amplitude 0.05m and period 12s. Find
   a) speed as it passes equilibrium position
   b) maximum acceleration of the particle

Solution

a)speed at equilibrium

$$v_{max} = \omega r$$

$$v_{max} = \frac{2\pi}{T} r$$

$$v_{max} = \frac{2x\frac{22}{7}x0.05}{12} = 0.026ms^{-1}$$

b) $a_{max} = \omega^2 r$

$$a_{max} = \left(\frac{2\pi}{T}\right)^2 r$$

$$a_{max} = \left(\frac{2x\frac{22}{7}}{12}\right)^2 x\ 0.05$$

$$a_{max} = 0.014ms^{-2}$$

9.2.0: TO SHOW A GIVEN MOTION IS SIMPLE HARMONIC

This requires to show that a particular motion has an acceleration of the form [$a = -\omega^2 x$.] and find the period (T= $\frac{2\pi}{T}$)

Steps

i) Identify the forces acting on the body at equilibrium position

ii) Identify the forces acting on the body after displacement from the equilibrium position

iii) Obtain an expression for the restoring force after the displacement and equate this restoring force to [ma] in accordance with Newton's second law of motion.

iv) Compare the expression got with basic standard expression for S.H.M. if it's comparable to ($a = -\omega^2 x$), then motion is simple harmonic.

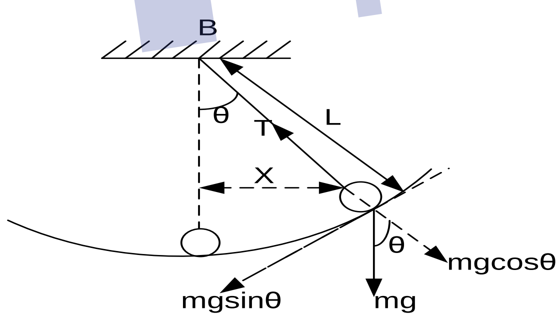
Examples of S.H.M

9.2.1: SIMPLE PENDULUM

Consider a mass m suspended by a light inelastic string of length L from a fixed point B.

At equilibrium the bob lies in a vertical plane with the tension in the string being balanced by the weight of the bob.

If the bob is given a small vertical displacement through an angel θ and released, we show that a bob moves with simple harmonic motion



Resolving tangentially, gives the restoring force

Restoring force = $-mgsin\theta$

By Newton's 2nd law $ma = -mgsin\theta$

$a = -gsin\theta$ ……………………………..2

If the displacement is small, then θ is very small such that the arc length is equal to x.

$$Sin\theta \approx \theta \approx \frac{x}{l}$$

$$a = -g\theta$$

$$a = -g\frac{x}{l}$$

$$a = -\left(\frac{g}{l}\right)x$$

it is in the form $a = -\omega^2 x$ and hence performs S.H.M with period

$$\omega^2 = \frac{g}{l}$$

$$\boxed{\omega = \sqrt{\frac{g}{l}}}$$

But $\omega = \frac{2\pi}{T}$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}}$$

9.2.1: Determination of acceleration due to gravity (g) using a simple pendulum

- Starting with a measured length L of the pendulum, the pendulum is clamped between 2 wood pieces from a retort stand.
- A bob is then given a small angular displacement from the vertical position and released.
- The time t for 20 oscillation is obtained, find period T and henceT^2
- Repeat the procedure for different values of length of the string.
- A graph of T^2 against L is then drawn and its slope S calculated.

Hence acceleration due to gravity is obtained from $g = \frac{4\pi^2}{s}$

Factors which affect the accuracy of g when using a simple pendulum.

1. The nature of the string. The string should be inelastic
2. Air resistance (dissipative force). In present of air the motion of a simple pendulum is highly damped such that the oscillation dies out quickly that affecting the period.
3. The displacement of the bob from the equilibrium position should be small such that the oscillation remain uniform.
4. The mass of the bob should be small to minimize the effect of dimension of the object.
5. In accuracies in the timing and measuring extensions.

Examples ;

A bob of a simple pendulum moves simple harmonically with amplitude 8.0cm and period 2.00s. its mass is 0.50kg, the motion of the bob is un damped. Calculate maximum values of;

a) The speed of the bob, and

b) The kinetic energy of the bob.

Solution

a) m=0.5kg , r=8cm=0.08m, T=2s

$v_{max} = \omega r$

$v_{max} = \frac{2\pi}{T} r = \frac{2x\frac{22}{7}}{2} x0.08$

$v_{max} = 0.25ms^{-1}$

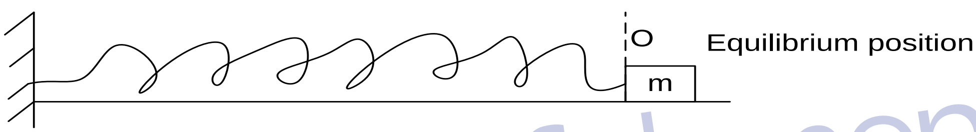
b) $K.E_{max} = ½\, m\, v_{max}^2$

$K.E_{max} = ½\, x0.5x(0.25)^2 = 1.563x10^{-2}J$

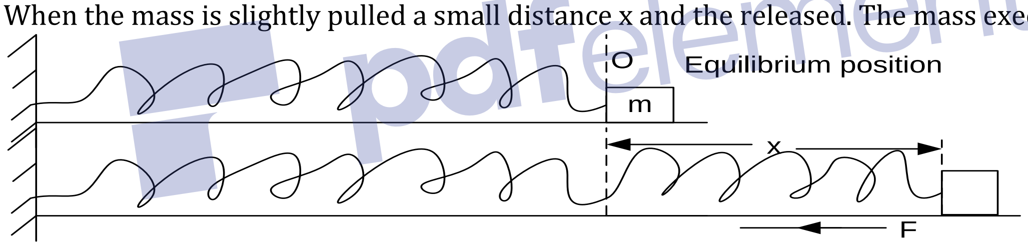
MASS ON A SRING

a) A horizontal spring attached to a mass

Consider a spring lying on a smooth horizontal surface in which one end of the spring is fixed and the other end attached to a particle of mass m



When the mass is slightly pulled a small distance x and the released. The mass executes S.H.M



The restoring force F is given by hooke's law

$F = -kx$-- (1)

By Newton's 2nd law

$F = ma$ --------------------------------------- (2)

$ma = -kx$

a= $-\left(\frac{k}{m}\right) x$-------------------------------- (3)

Where k is the spring constant

Equation (3) is in the form$[a = -\omega^2 x\,]$, therefore the body performs S.H.M

$\therefore \omega^2 = \frac{k}{m}$

=> $\boxed{\omega = \sqrt{\frac{k}{m}}}$

But $\omega = \frac{2\pi}{T}$

$\frac{4\,\pi^2}{T^2} = \frac{k}{m}$

$T^2 = \frac{4\,\pi^2 m}{k}$

$$\boxed{T = 2\pi\sqrt{\frac{k}{m}}}$$

Also f = $\frac{1}{T}$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$$

Example : UNEB 2011 No 4C

A horizontal spring of force constant 200 Nm^{-1} is fixed at one end and a mass of 2kg attached to the free end and resting on a smooth horizontal surface. The mass is pulled through a distance of 4.0cm and released. Calculate the;

i) Angular speed

ii) Maximum velocity attained by the vibrating body, acceleration when the body is half way towards the centre from its initial position.

Solution

i) From $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200}{2}} = 10$rads^{-1}

ii) $v_{max} = \omega r$

$v_{max} = 10x\frac{4}{100}$ =0.4ms^{-1}

Note: the small distance pulled and released becomes the amplitude

$a = -\omega^2 x$

where its half towards the centre

$x = \frac{r}{2}$

$x = \frac{4x10^{-2}}{2}$

$a = -\omega^2 x = 10^2 x\frac{4x10^{-2}}{2} = 2ms^{-2}$

Alternatively

$F = ma$

$F = kx$

$k\frac{r}{2} = ma$

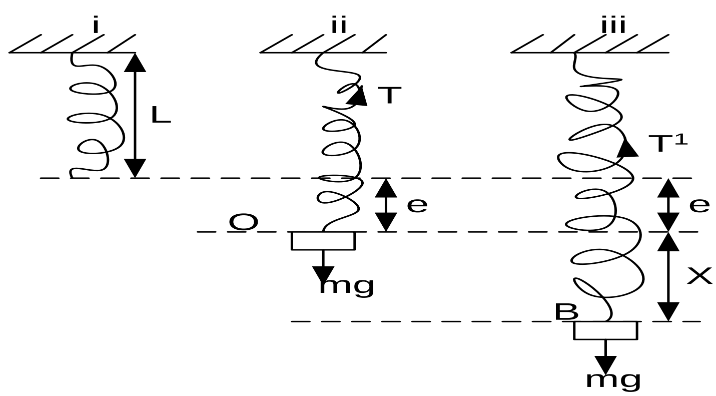
$a = \frac{200x4x10^{-2}}{2x2} = 2ms^{-2}$

b) Oscillation of mass suspended on a helical spring

Consider a helical spring or elastic string suspended from a fixed point.

When a mass is attached to the spring, it stretches by length, e and comes to equilibrium positions 0.

When the mass is pulled down a small distance, x and released the motion will be simple harmonic.



In position (ii) the mass is in equilibrium position

$T = mg$

And by hooke's law $T = ke$

$mg = ke$ -------------------------(1)

In position (iii) after displacement through x

The restoring fore is $= mg - T^1$

But by hooke's law $T^1 = k(e + x)$

Restoring force $= mg - k(e + x)$

By Newton's 2nd law $mg - k(e + x) = ma$

But from equation 1 $mg = ke$

$ke - k(e + x) = ma$

$ke - ke - kx = ma$

$-kx = ma$

$a = -\frac{k}{m}x$ -------------------------------- [3]

Equation 3 is in the form $a = -\omega^2 x$ and therefore performs S.H.M

$\omega^2 = \frac{k}{m}$

$\boxed{\omega = \sqrt{\frac{k}{m}}}$ ---------------------------------- [4]

But $\omega = \frac{2\pi}{T}$

$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$

$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$

Note:

From [1] $mg = ke$

$\frac{k}{m} = \frac{g}{e}$

$\omega = \sqrt{\frac{g}{e}}$

$T = 2\pi\sqrt{\frac{e}{g}}$

$f = \frac{1}{2\pi}\sqrt{\frac{g}{e}}$

9.2.2: Determination of acceleration due to gravity using a vertically loaded spring

- Clamp a spring with pointer vertically besides a meter rule, note and record the initial pointer reading; P_o
- Suspend a known mass on the spring and note and record the new reading P of the pointer on the meter rule
- The extension e of the spring is obtained from e= $P - P_0$
- Give the mas a small vertical displacement and obtain the time t for 20 oscillations. Find the period T and T^2
- Repeat the procedure for different values of the masses.
- Plot a graph of **T^2** against **e** and find the slope, s of the graph

Hence acceleration due to gravity is determined from $g = \frac{4\pi^2}{s}$

Examples

1. A 100g mass is suspended vertically from a light helical spring and the extension in equilibrium is found to be 10cm. The mass is now pulled down a further 0.5cm and it is released from rest.
    i) Show that the subsequent motion is simple harmonic

ii) Find the period of oscillation

iii) What is the maximum kinetic energy of the mass

Solution

$m = 100g = 0.1kg,$

$e = 10cm = 0.1m,$

$r = 0.5cm = 0.005m$

From $\omega = \sqrt{\frac{k}{m}}$

$T = 2\pi\sqrt{\frac{m}{k}}$

But also $mg = ke$

Therefore $\frac{k}{m} = \frac{g}{e}$

$T = 2\pi\sqrt{\frac{e}{g}}$

$T = 2\pi\sqrt{\frac{0.1}{9.81}} = 0.63s$

$v_{max} = \omega r$

$v_{max} = \frac{2\pi}{T}r = \frac{2x\frac{22}{7}}{0.63}x0.05$

$v_{max} = 0.0499ms^{-1}$

$K.E_{max} = ½\, m\, v_{max}^2$

$= ½\, x0.1x(0.0499)^2$

$K.E_{max} = 1.245x10^{-4}J$

2. A mass hangs from a light spring. The mss is pulled down 30mm from its equilibrium position and then released from rest. The frequency of oscillation is 0.5Hz. calculate
   a) The angular frequency, ω of the osculation
   b) The magnitude of the acceleration at the instant it is released from rest

Solution

Distance pulled down ward and released becomes the amplitude

$\therefore r = 30mm = 30x10^{-3}m$

$f = 0.5Hz$

a) Angular frequency ω

$\omega = 2\pi f$

$\omega = 2\pi x0.5$

$\omega = \pi\ rad\ s^{-1}$

$\omega = 3.14rad\ s^{-1}$

b) When it is released from rest the displacement is equals to amplitude and the acceleration is maximum.

$a_{max} = \omega^2 r$

$a_{max} = (3.14)^2 x30x10^{-3}$

$a_{max} = 0.296ms^{-2}$

Exercise:**23**

1. When a metal cylinder of mass 0.2kg is attached to the lower end of a light helical spring, the upper end of which is fixed, the spring extends by 0.16m. the metal cylinder is then pulled down a further 0.08m.
   i) Find the force that must be exerted to keep it there. **An [1.0N]**
   ii) The cylinder is then released. Find the period of vertical oscillation and the kinetic energy the cylinder posses when it passes through its mean position. **An[0.79s, 0.04J]**

2. A mass of 0.2kg is attached to the lower end of a helical spring and produces extension of 5.0cm. The mass is now pulled down at a further distance and released. Calculate
    a) the force constant of the spring
    b) The period of the subsequent motion
    c) The maximum value of the acceleration during the motion

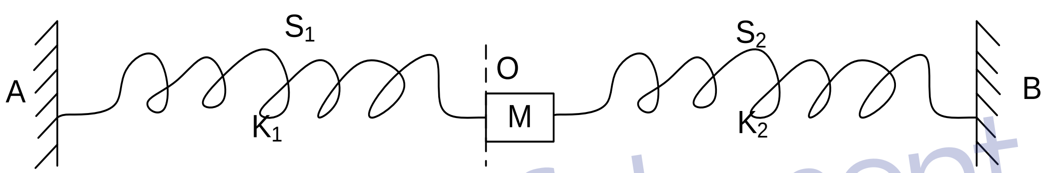
    An[39.24Nm^{-1}, 0.45s, 3.924ms^{-2}]

COMBINED SPRINGS

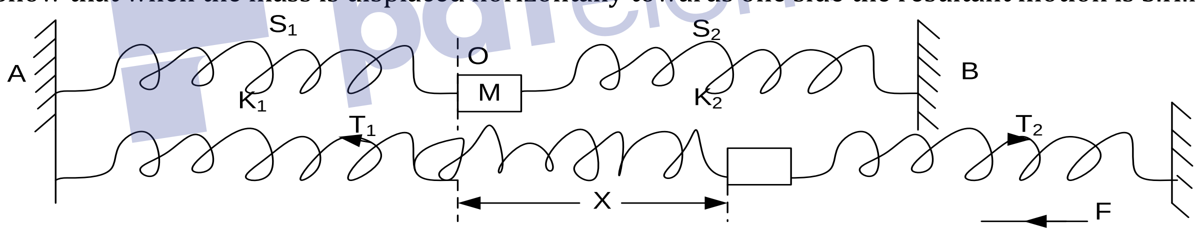
a) horizontal springs

9.2.3: TWO HORIZONTAL SPRINGS WITH A MASS BETWEEN THEM

Consider two springs with spring constants K_1, and K_2 attached to fixed points and mass attached between them.



Show that when the mass is displaced horizontally towards one side the resultant motion is S.H.M



Extension of $S_1 = x$

Compression of $S_2 = x$

Restoring force $F = -(T_1 + T_2)$

But by Hooke's law

$T_1 = k_1 x \text{ and } T_2 = k_2 x$

$F = -(k_1 x + k_2 x)$

$F = -(k_1 + k_2)x$ --------------------------- (1)

By Newton's 2nd law

$F = ma$ -------------------------------------- (2)

$ma = -(k_1 + k_2)x$

$a = -\left(\frac{k_1 + k_2}{m}\right)x$ ---------------------------- (3)

Equation 3 in the form $a = -\omega^2 x$ and therefore it performs S.H.M

$$\omega^2 = \left(\frac{k_1 + k_2}{m}\right)$$

$$\boxed{\omega = \sqrt{\left(\frac{k_1 + k_2}{m}\right)}} \text{ ----------------------------- (4)}$$

But $\omega = \frac{2\pi}{T}$

$$\frac{2\pi}{T} = \sqrt{\left(\frac{k_1 + k_2}{m}\right)}$$

$$T = \frac{2\pi}{\sqrt{\left(\frac{k_1 + k_2}{m}\right)}}$$

$$\boxed{T = 2\pi\sqrt{\left(\frac{m}{k_1 + k_2}\right)}}$$

$$f = \frac{1}{T}$$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\left(\frac{k_1 + k_2}{m}\right)}}$$

Note: when the springs are identical

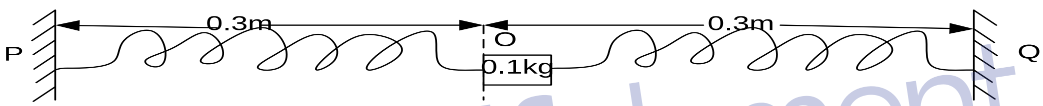
$$k_1 = k_2 = k$$

$$T = 2\pi\sqrt{\left(\frac{m}{2k}\right)}$$

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{2k}{m}\right)}$$

Example

1. A mass of 0.1kg is placed on a frictionless horizontal surface and connected to two identical springs of negligible mass and a spring constant of 33.5Nm^{-1}. The springs are then attacked to fixed point p and Q on the surface as shown below.



The mass is given a small displacement along the line of the spring and released

i) Show that the system will execute S.H.M

ii) Calculate the period of oscillation

iii) If the amplitude of oscillation is 0.05m, calculate the maximum kinetic of the system.

Solution

ii) From $T = 2\pi\sqrt{\left(\frac{m}{k_1 + k_2}\right)}$

$$T = 2x\frac{22}{7}\sqrt{\left(\frac{0.1}{33.5 + 33.5}\right)}$$

$$T = 0.243s$$

iii) r=0.05m

$$v_{max} = \omega r$$

$$v_{max} = \frac{2\pi}{T}r$$

$$= \frac{2x\frac{22}{7}}{0.243}x0.05$$

$$v_{max} = 1.293ms^{-1}$$

$$K.E_{max} = ½\, m\, v_{max}^2$$

$$= ½\, x0.1x(1.293)^2$$

$$K.E_{max} = 0.084J$$

UNEB 1998 No 3

2. A body of mass 4kg rests on a smooth horizontal surface. Attached to the body are two pieces of light elastic strings each of length of 1.2m and force constant 6.25Nm^{-1}. The ends are fixed to two

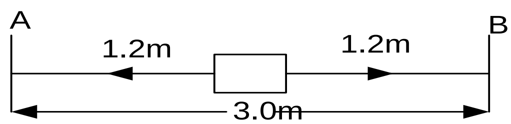
points A and B 3.0m apart as shown in the figure below. The body is then pulled through 0.1m towards B and then released.

i) Show that the body executes S.H.M

ii) Find the period of oscillation of the body

iii) Calculate the speed of the body when it is 0.03m from the equilibrium position

Solution



ii) From $T = 2\pi\sqrt{\left(\frac{m}{k_1 + k_2}\right)}$

$$T = 2x\frac{22}{7}\sqrt{\left(\frac{4}{6.25 + 6.25}\right)}$$

T= 3.55s

iii) $v^2 = \omega^2(r^2 - x^2)$

Amplitude $r = 0.1\ m$

$x = 0.03m$

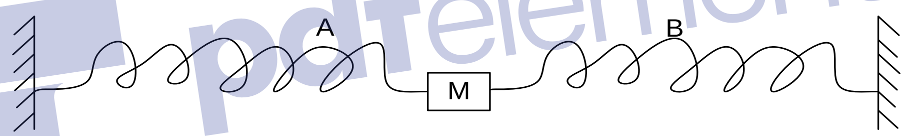
$$\omega = \frac{2\pi}{T}$$

$$v^2 = \left(\frac{2\pi}{T}\right)^2(r^2 - x^2)$$

$$v^2 = \frac{4x3.14^2}{3.55^2}(0.1^2 - 0.03^2)$$

$$v = 0.169m/s$$

3. The figure below shows a mass of 200g resting on a smooth horizontal table, attached to two springs A and B of force constants k_1 and k_2 respectively



The block is pulled through a distance of 8cm to the right and then released.

(i) Show that the mass oscillates with simple harmonic motion and find the frequency of oscillation if $k_1 = 120Nm^{-1}$ and $k_2 = 200Nm^{-1}$

(ii) Find the new amplitude of oscillation when a mass of 120g is dropped vertically onto the block as the block passes the equilibrium position. Assume that the mass sticks to the block

Solution

i) From $f = \frac{1}{2\pi}\sqrt{\left(\frac{k_1 + k_2}{m}\right)}$

$$f = {}^{1}\Big/\left(2x\frac{22}{7}\right)\sqrt{\left(\frac{60 + 100}{0.2}\right)}$$

$$f = 6.37Hz$$

ii) By conservation of momentum:

$$m_1u = (m_1 + m_2)v_{max}$$

$$v_{max} = \frac{m_1u}{(m_1 + m_2)}$$

$v_{max} = \omega^1r^1$ and $u_{wax} = \omega r$

$$\omega^1 = \sqrt{\left(\frac{k_1 + k_2}{m}\right)} = \sqrt{\left(\frac{120 + 200}{0.32}\right)} = 10\sqrt{10}rads^{-1}$$

$$\omega = \sqrt{\left(\frac{k_1 + k_2}{m}\right)} = \sqrt{\left(\frac{120 + 200}{0.2}\right)} = 40rads^{-1}$$

$$u_{wax} = \omega r = 40x0.08 = 3.2m/s$$

$$\therefore\ v_{max} = \frac{m_1 u}{(m_1 + m_2)}$$

$$10\sqrt{10}\, r^1 = \frac{0.2x3.2}{(0.2 + 0.12)}$$

$$r^1 = 0.632m$$

EXERCISE:24

A block of mass 0.1kg resting on a smooth horizontal surface and attached to two springs s_1 and s_2 of force constant 60Nm^{-1} and 100Nm^{-1}respectively. The block is pulled a distance of 4x10^{-2}m to the right and the released.

i) Show that the mass executes S.H.M and fixed the frequency of oscillation

ii) Find the new amplitude of oscillation when the block is added a mass of 0.06kg on top as the block passes the equilibrium position. **An (6.4 Hz, 0.032m)**

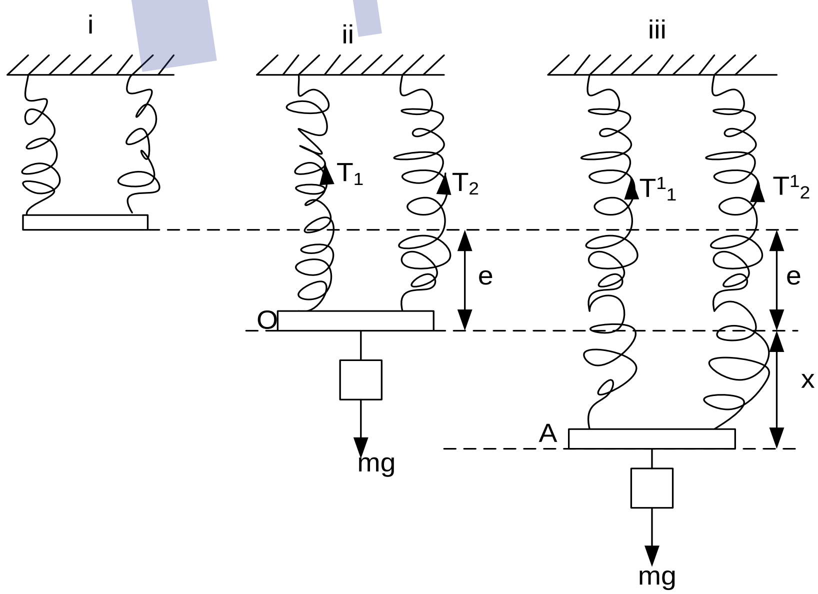
b) Vertical springs

9.2.4: Vertically loaded springs in parallel

Consider two springs of force constants k_1 and k_2 suspended from the same rigid support side by side. When a mass is attached to the mid point of a rod connected to the lower ends of the springs.

The system rests in equilibrium

When the mass is displaced a small distance vertically downwards and then released the system execute S.H.M



At equilibrium : $Mg\ = T_1 + T_2$

But by Hooked law $T_1 = k_1 e$ and$T_2 = k_2 e$

$mg = (\,k_1 + k_2)$ e------------------------[1]

When the mass is displaced then:

Restoring force $= mg - (T_1{}^1 + T_2{}^1)$

But by Hooke's law

$T_1{}^1 = k_1(e+x)$ and $T_2{}^1 = k_2(e+x)$

Restoring force $= mg - [\,k_1(e+x) + k_2(e+x)]$

By Newton's second law Restoring force $= ma$

$mg - [\,k_1(e+x) + k_2(e+x)] = ma$ -- (2)

But from equation 1 $mg = (\,k_1 + k_2)e$

$(k_1 + k_2)e - [\,k_1(e+x) + k_2(e+x)] = ma$

$-k_1x - k_2x = ma$

$-(\,k_1x + k_2x) = ma$

$a = -\left(\frac{k_1 + k_2}{m}\right)x$ ------------------------ (3)

Equation 3 is in the form $a = -\omega^2 x$ and therefore performs S.H.M

$\omega^2 = \left(\frac{k_1 + k_2}{m}\right)$

$\boxed{\omega = \sqrt{\left(\frac{k_1 + k_2}{m}\right)}}$ ------------------------ (4)

T=$\frac{2\pi}{\omega}$

Period T=$2\pi\sqrt{\left(\frac{m}{k_1+k_2}\right)}$

$f = \frac{1}{2\pi}\sqrt{\left(\frac{k_1 + k_2}{m}\right)}$

Note: From equation 1

$mg = (\,k_1 + k_2)e$

$\frac{m}{(\,k_1+k_2)} = \frac{e}{g}$

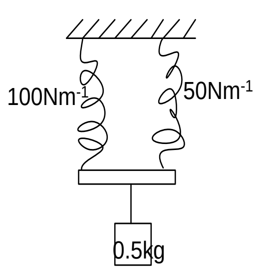
$\omega = \sqrt{\frac{g}{e}}$

T=$2\pi\sqrt{\left(\frac{e}{g}\right)}$

$f = \frac{1}{2\pi}\sqrt{\frac{g}{e}}$

Examples

1. A mass of 0.5kg is suspended from the free ends of two springs of force constant 100Nm^{-1} and 50Nm^{-1} respectively as shown in the figure below.



Calculate ;

i) The extension produced

ii) Tension in each string

iii) Energy stored in the string

iv) Frequency of small oscillations when he mass is given a small vertical displacement

Solution

i) At equilibrium $mg = (k_1 + k_2)$ e

$$e = \frac{mg}{k_1 + k_2} = \frac{0.5x9.81}{100 + 50} = 0.0327m$$

ii) Tension in each string

From Hooke's law $T_1 = k_1 e$

$$T_1 = 100x0.0327 = 3.27N$$

Also $T_2 = k_2 e = 50x0.0327 = 1.635N$

iii) Energy stored is always stored as elastic potential energy of the spring

$$P.E_{Elatic} = \frac{1}{2}ke^2$$

$$E_1 = \frac{1}{2}k_1e^2 = \frac{1}{2}x\ 100\ x\ (0.0327)^2 = 0.0535J$$

$$E_2 = \frac{1}{2}k_2e^2 = \frac{1}{2}x\ 50\ x\ (0.0327)^2 = 0.0267J$$

$$P.E_{Elatic} = E_1 + E_2$$

$$P.E_{Elatic} = 0.0535 + 0.026$$

$$P.E_{Elatic} = 0.0802J$$

iv) Frequency

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{e}}$$

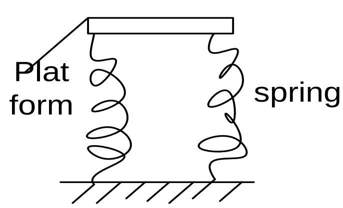
$$f = \left(\frac{1}{2x\frac{22}{7}}\right)\sqrt{\frac{9.81}{0.0327}} = 2.757Hz$$

Alternatively

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{k_1 + k_2}{m}\right)}$$

$$f = \left(\frac{1}{2x\frac{22}{7}}\right)\sqrt{\left(\frac{100 + 50}{0.5}\right)} = 2.757Hz$$

2. A light platform is supported by tow identical springs each having spring constants 20Nm^{-1} as shown below.



a) Calculate the weight which must be placed on the centre of the platform in order to produce a displacement of 3.0cm.

b) The weight remains on the platform and the platform is depressed a further 1.0cm and then released

i) What is the frequency of the oscillation

ii) What is the maximum acceleration of the platform

Solution

a) Compression $e = 3.0cm = 0.03m$

At equilibrium

$mg = T_1 + T_2$

$mg = (k_1 + k_2)$ e

$mg = (20 + 20)x0.03$

$mg = 1.2N$

$weight = 1.2N$

b) Amplitude r=1.0cm=0.01m

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{e}}$$

$$f = \left(\frac{1}{2x\frac{22}{7}}\right)\sqrt{\frac{9.81}{0.03}}$$

$f = 2.89Hz$

$a_{max} = \omega^2 r$

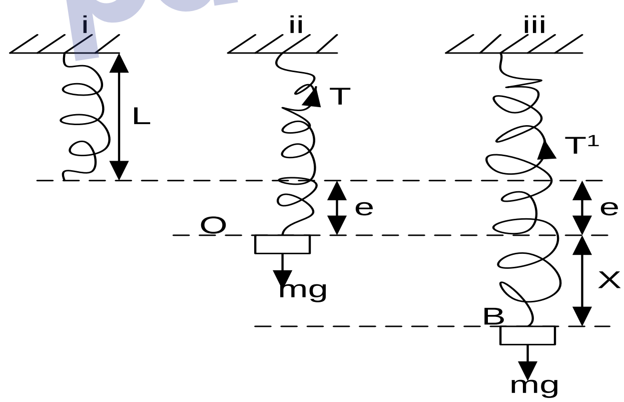
$a_{max} = (2\pi f)^2 r$

$$a_{max} = \left(2x\frac{22}{7}x2.89\right)^2 x0.01$$

$a_{max} = 3.297ms^{-2}$

9.2.5: Vertically loaded spring in series

Consider two springs of constants k_1 and k_2 suspended in series, mass m is then attached to the lower end of the last spring such that at equilibrium each spring extends by e_1 and e_2 respectively.



The springs are assumed to be light such that they have the same tension

Let e be extension in the combination

At equilibrium $mg = T$

$mg = ke$ ----------------------------------- (1)

Where k is the combined spring constant

$k = \frac{k_1 k_2}{k_1 + k_2}$ for series connection

After a small displacement, the restoring force

$mg - T^1 = ma$

$mg - k(e + x) = ma$

$mg - ke - kx = ma$ -------------------- [2]

but from equation[1] $mg = ke$

$ke - ke - kx = ma$

$-kx = ma$

$a = -\left(\frac{k}{m}\right)x$--------------------------------[3]

it is in the form $a = -\omega^2 x$

$$\therefore \ \omega^2 = \frac{k}{m}$$

But $k = \frac{k_1 k_2}{k_1 + k_2}$

$$\omega^2 = \frac{k_1 k_2}{k_1 + k_2} \Big/ m$$

$$\boxed{\omega = \sqrt{\frac{k_1 k_2}{k_1 + k_2} \Big/ m}}$$ ------------------------- [4]

$$T = 2\pi\sqrt{\frac{(k_1 + k_2)m}{k_1 k_2}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{k_1 + k_2} \Big/ m}$$

Note

The tension is the same in both springs

$mg = T$

$mg = ke$

$\therefore mg = k_1 e_1$ and $mg = k_2 e_2$

$e_1 = \frac{mg}{k_1}$ and $e_2 = \frac{mg}{k_2}$

but $e = e_1 + e_2$

$$e = \frac{mg}{k_1} + \frac{mg}{k_2}$$

$$e = mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$e = mg\left(\frac{k_1 + k_2}{k_1 k_2}\right)$$

$$e\left(\frac{k_1 k_2}{k_1 + k_2}\right) = mg$$

$\therefore ek = mg$

$mg = ke$

$\therefore k = \frac{k_1 k_2}{k_1 + k_2}$

Also

$$\omega = \sqrt{\frac{g}{e}}$$

$$T = 2\pi\sqrt{\left(\frac{e}{g}\right)}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{e}}$$

Example

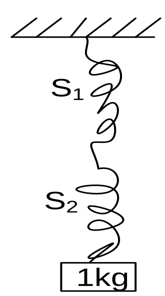
UNEB 2004 No 3b

A mass of 1.0kg is hung from two springs S_1 and S_2 connected in series as shown

The force constant of the springs are $100 Nm^{-1}$ and $200 Nm^{-1}$ respectively. Find

i) The extension produced in the combination

ii) The frequency of oscillation of the mass if it is pulled downwards and released



Solution

m=1kg , k_1=100Nm^{-1}, k_2=200 Nm^{-1}

At equilibrium $\quad mg = ke$

$e = \frac{mg}{k}$ $\quad$ but k= $\frac{k_1 k_2}{k_1 + k_2}$

$e = {mg}\Big/{\frac{k_1k_2}{k_1+k_2}}$

$e = 1x9.81 \Big/ \left(\frac{100x200}{100+200}\right)$

$e = 0.1472m$

ii)

$$f = \frac{1}{2\pi}\sqrt{\frac{k_1k_2}{k_1+k_2}\Big/m}$$

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{100x200}{100+200}\right)\Big/1}$$

f=1.299Hz

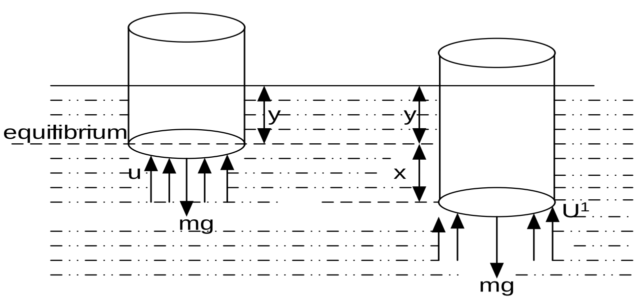
NB: For all S.H.M, the following assumptions hold

1- displacement from equilibrium position is small such that Hooke's law is obeyed throughout the motion

2- no dissipative forces act

9.2.6: S.H.M OF A FLOATING CYLINDER

Consider a uniform cylindrical rod of length L and cross sectional area A and density, ρ floating vertically in a liquid of density, δ. When the rod is given a small downward displacement x and released, the rod executes S.H.M.



At equilibrium, up thrust of the liquid on the rod is equal to the weight of the displaced fluid

U =weight of the liquid is displaced

U =mass of liquid displaced x g

U = volume of liquid displaced x density x g

$U = Ay\delta g$ --------------------------------------- [1]

When the rod is given a downward displaced x, the new up thrust is U^1

U^1 =weight of the liquid is displaced

=mass of liquid displaced x g

U^1 = volume of liquid displaced x density x g

$U^1 = A(y+x)\delta g$ ---------------------------- [2]

On release, the restoring force on the rod is

$U - U^1 = ma$

$Ay\delta g - A(y+x)\delta g = ma$

$-A\delta g\, x = ma$

$a = -\left(\frac{A\delta g}{m}\right)x$

m is mass of cylinder =volume of cylinder x density of cylinder

$m = Al\rho$

$$a = -\left(\frac{A\delta g}{A\,l\,\rho}\right)x$$

$$a = -\left(\frac{\delta g}{l\,\rho}\right)x \text{-------------------------- [3]}$$

it is the form $a = -\omega^2 x$

$$\omega^2 = \frac{\delta g}{l\,\rho}$$

$$\boxed{\omega = \sqrt{\frac{\delta g}{l\,\rho}}} \text{---------------------------- [4]}$$

$$T = 2\pi\sqrt{\left(\frac{\rho l}{\delta g}\right)}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{\delta g}{l\,\rho}}$$

Examples : UNEB 2000 No2b

1. A Uniform cylindrical rod of length 8cm, cross sectional area 0.02m² and density 900kgm⁻³ floats vertically in a liquid of density 1000kgm⁻³. The rod is depressed through a distance of 0.005m and then released.

i) Show that the rod performs S.H.M (5mk)

ii) Find the frequency of the resultant oscillation (4mk)

iii) Find the velocity of the rod when it is at a distance of 0.004m above the equilibrium position

Solution

ii) $f = \frac{1}{2\pi}\sqrt{\frac{\delta g}{l\,\rho}}$

$$f = \left(\frac{1}{2x\frac{22}{7}}\right)\sqrt{\frac{1000x9.81}{8x10^{-2}x900}}$$

$f = 1.858Hz$

iii) $v^2 = \omega^2(r^2 - x^2)$

r=0.005m , $x = 0.004m$, $\omega = 2\pi f$

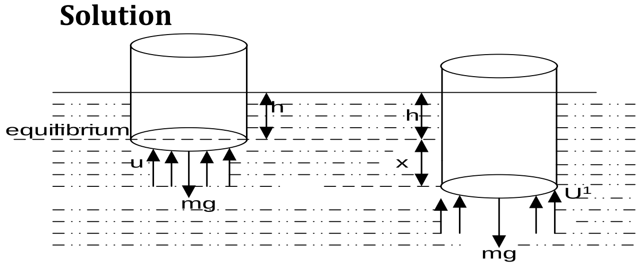
$$v^2 = (2\pi f)^2(r^2 - x^2)$$

$$v^2 = \left(2x\frac{22}{7}x1.858\right)^2(0.005^2 - 0.004^2)$$

$$v = 3.5x10^{-2}ms^{-1}$$

2. A wooden rod of uniform cross sectional area A floats with a height h immersed in a liquid of densityδ. The rod is given a slight downward displacement and released. Show that the resulting motion is S.H.M with a time period of $2\pi\sqrt{\frac{h}{g}}$

Solution



At equilibrium, upthrust of the liquid on the rod is equal to the weight of the displaced fluid

U =weight of the liquid is displaced

=mass of liquid displaced x g

U = volume of liquid displaced x density x g

$U = Ah\delta g$ ------------------------------------ [1]

When the rod is given a downward displaced x, the new up thrust is U¹

U^1 =weight of the liquid is displaced

=mass of liquid displaced x g

U^1 = volume of liquid displaced x density x g

$U^1 = A(h+x)\delta g$ ----------------------------- [2]

On release, the restoring force on the rod is

$U - U^1 = ma$

$Ah\delta g - A(h+x)\delta g = ma$

$-A\delta g\, x = ma$

$a = -\left(\frac{A\delta g}{m}\right)x$

m is mass of cylinder =volume of cylinder x density of cylinder

$m = Al\delta$

$a = -\left(\frac{A\delta g}{A\,h\,\delta}\right)x$

$a = -\left(\frac{g}{h}\right)x$-------------------------- [3]

it is the form $a = -\omega^2 x$

$\omega^2 = \frac{g}{h}$

$\boxed{\omega = \sqrt{\frac{g}{h}}}$---------------------------- [4]

$T = \frac{2\pi}{\omega}$

$T=2\pi\sqrt{\left(\frac{h}{g}\right)}$

Example

A cylindrical test tube of thin wall and mass 1kg with a piece of lead of mass 1kg fixed at its inside bottom floats vertically in the liquid.

When the test tube is slightly depressed and released it oscillates vertically with a period of one second ($T = 1s$).

If some extra copper beads are put in the test tube, it floats vertically with a period of 1.5 seconds. Find the mass of the copper beads in the test tube.

Solution

$T=2\pi\sqrt{\left(\frac{h}{g}\right)}$

$1=2\pi\sqrt{\left(\frac{h_1}{9.81}\right)}$

$h_1 = \frac{1^2 x 9.81}{4\pi^2}$ =0.2485m

Also $1.5=2\pi\sqrt{\left(\frac{h_2}{9.81}\right)}$

$h_2 = \frac{1.5^2 x 9.81}{4\pi^2}$ =0.5591m

at equilibrium U =weight of liquid displaced

$2g = \text{A}\, h_1\delta\, g$

$2 = \text{A}\, h_1\delta$ ---------------------------- (1)

Also when a mass m is added

$(2+m)g = \text{A}\, h_2\delta\, g$

$(2+m) = \text{A}\, h_2\delta$ -------------------- (2)

Equation 2 divided by equation 1

$\frac{(2+m)}{2} = \frac{\text{A}\, h_2\delta}{\text{A}\, h_1\delta}$

$\frac{(2+m)}{2} = \frac{h_2}{h_1}$

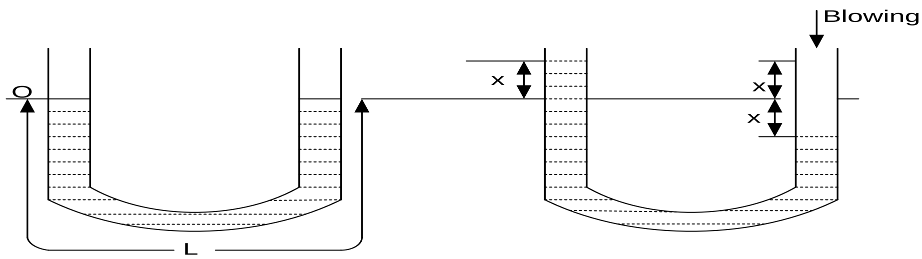
$m = \frac{2\ x\ h_2}{h_1} - 2$

$m = \frac{2\ x\ 0.5591}{0.2485} - 2$

$m = 2.5kg$

9.2.7: A LIQUID OSCILLATING IN A U-TUBE

Consider a column of liquid of density σ and total length l in a U-tube of uniform cross sectional area A. Suppose the level of the liquid on the right side is depressed by blowing gently down that side, the levels of liquid will oscillate for a short time about their respective or equilibrium positions O.



When the meniscus is at a distance, x , from equilibrium position, a differential height of liquid of, $2x$, is produced

Excess pressure on liquid $= 2x\delta g$ from $[p = h\delta g]$

Force on liquid =pressure x Area $= 2x\delta gA$

Restoring force $= -2x\delta gA$ ---------------------------------[1]

Newton's 2nd law : $ma \ = -2x\delta gA$

$a = -\left(\frac{2\delta gA}{m}\right)x$ ---[2]

But mass of liquid in the tube =volume of liquid x $\delta = Al\delta$

$$a = -\left(\frac{2\delta gA}{A\,l\,\delta}\right)x$$

$a = -\left(\frac{2g}{l}\right)x$--- [3]

it is in the form $a = -\omega^2 x$

$$\omega^2 = \frac{2\,g}{l}$$

$$\boxed{\omega = \sqrt{\frac{2\,g}{l}}}$$

$$T \ = \frac{2\pi}{\omega}$$

$$\boxed{T \ = \ 2\pi\sqrt{\frac{l}{2\,g}}}$$

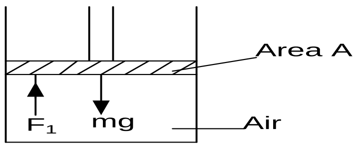
9.2.8:S.H.M IN A FRICTIONLESS AIR TIGHT PISTON

A volume v of air and pressure p is contained in a cylindrical vessel of cross section area A by frictionless air tight piston of mass m.

Show that on slight forcing down the piston and then releasing it, the piston will exert S.H.M given by

$$T = \frac{2\,\pi}{A}\sqrt{\frac{m\,v}{P}}$$

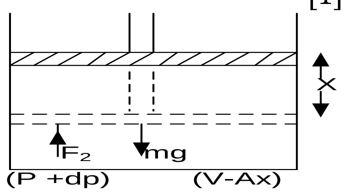
Solution



At Equilibrium

$F_1 = PA$

$PA = mg$ ------------------------------- [1]



When the piston is given a slight downward displaced x, the volume decrease to

$[v - dv]$

where $dv = Ax$

$F_2 = \ (P + dp)A$

On releasing, the restoring force

$= (P + dp)A - mg$

But by Bewton's 2nd law

$ma = -[\ (P + dp)A - mg]$

$ma = -[\ PA + Adp - mg]$

from Equation 1 $PA = mg$

$ma = -[mg + Adp - mg]$

$ma = \ -Adp$ -------------------------- [2]

If the displacement x is small, the compression will be almost isothermal obeying Boyle's law.

$[P_1V_1 = P_2V_2]$

$(P + dp)(v - Ax) = Pv$

$Pv - PAx + vdp - Axdp = Pv$

For small displacement $Axdp \approx 0$

$Pv - PAx + vdp - 0 = Pv$

$PAx = vdp$

$dp \ = \frac{PAx}{v}$

put into equation 2

$ma = -A\left(\frac{PAx}{v}\right)$

$a = -\left(\frac{PA^2}{m\,v}.\right)x$

it is in the form $a = -\omega^2 x$

$\omega^2 = \frac{PA^2}{m\,v}.$

$\omega = \sqrt{\frac{PA^2}{m\,v}}.$

$\omega = A\sqrt{\frac{P}{m\,v}}.$

But $T \ = \frac{2\pi}{\omega}$

T=$\frac{2\pi}{A}\sqrt{\left(\frac{mv}{p}\right)}$

$f = \frac{1}{T}$

$f = \frac{A}{2\pi}\sqrt{\frac{P}{m\,v}}$

Example

A piston in a car engine performs S.H.M. The piston has a mass of 0.50kg and its amplitude of vibration is 45mm. the revolution counter in the car reads 750 revolutions per minute. Calculate the maximum force on the piston.

Solution

$r = 45mm = 45x10^{-3}m,\ m = \ 0.5kg$

$f = 750\ rev/min$

$f = \frac{750}{60} = 12.5rev/s$

But $a_{max} = \omega^2 r$

$\omega = 2\pi f$

$a_{max} = (2\pi f)^2 r$

$a_{max} = \left(2x\frac{22}{7}x12.5\right)^2 x12.5$

a_{max} =277.583ms^{-2}

$F_{max} = ma_{max}$

$F_{max} = 0.5x277.583$

$F_{max} = 138.792N$

9.3.0: ENERGY CHANGES IN S.H.M

- In S.H.M there's always an energy exchange. At maximum displacement , all the energy is elastic potential energy while at equilibrium point all the energy is kinetic energy

a) Kinetic energy

It's the energy possessed by a body due to its motion

K.E = ½ mv^2

K.E = ½ m$\omega^2(r^2 - x^2)$

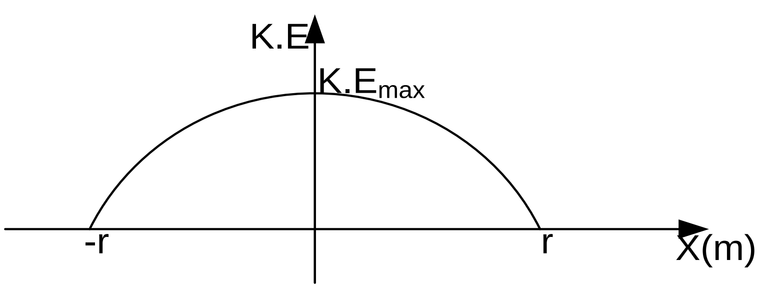
Note

i) The K.E is zero when the displacement x is equals to the amplitude

ii) K.E is maximum when the displacement x is zero

K.E_{max} = ½ m$\omega^2 r^2$

9.3.1: A graph of K.E against displacement



b) Elastic potential energy

This is the energy possessed by a body due to the nature of its particle i.e. compressed or stretched.

Force is applied to make particles stretch or compress and therefore the force does work, which work is stored in the body.

$\Delta w = F\Delta x$

But $F = kx$

$\Delta w = k\Delta x$

Total work done $\int_0^w dw = \int_0^x kx\, dx$

$w = \left[\frac{kx^2}{2}\right]_0^x$

$w = \frac{kx^2}{2}$

Elastic potential energy $= \frac{1}{2}kx^2$

Or

$\Delta w = F\Delta x$

But F= $m\omega^2$x

$\Delta w = m\omega^2 x \Delta x$

$$\int_0^w dw = \int_0^x m\omega^2 x\, dx$$

$W = ½\, m\omega^2 x^2$

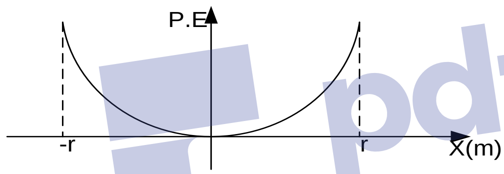
Elastic potential energy = ½ $m\omega^2 x^2$

Note :

i) Elastic potential energy is maximum when x is a maximum

ii) Elastic potential energy is zero when x=0 (equilibrium)

9.3.2: Graph of P.E against displacement



iii) Mechanical energy

This is the total energy possessed by a body due its motion and nature of its particles

$M.E = K.E + P.E$

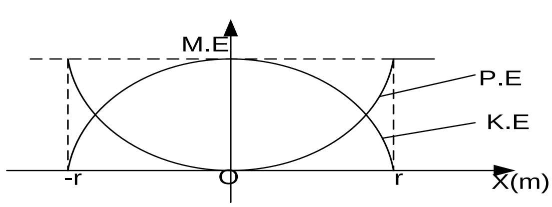
$=½\, m\omega^2(r^2 - x^2) + ½\, m\omega^2 x^2$

M.E = ½ $m\omega^2 r^2$

Note

Mechanical energy is constant

9.3.3: A graph of M.E against displacement



9.4.0: MECHANICAL OSCILLATION

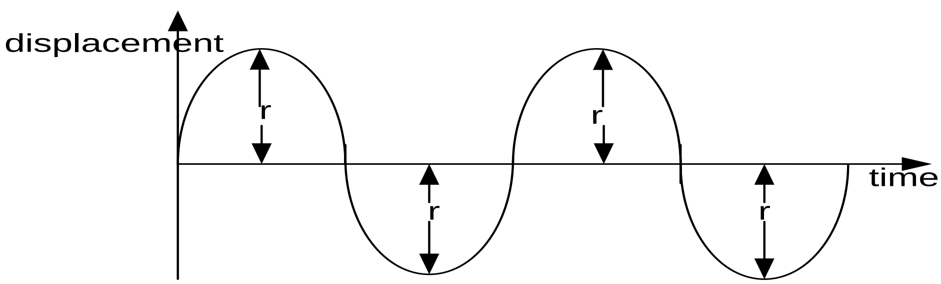
There are three types of oscillation i.e.

a) Free oscillation b) Damped oscillation c) Forced oscillation

a) Free oscillations

These are oscillations in which the amplitude remains constant and oscillating systems does not do work against dissipative force such as air friction, and viscous drag. Eg a pendulum bob in a vacuum

Displacement- time graph



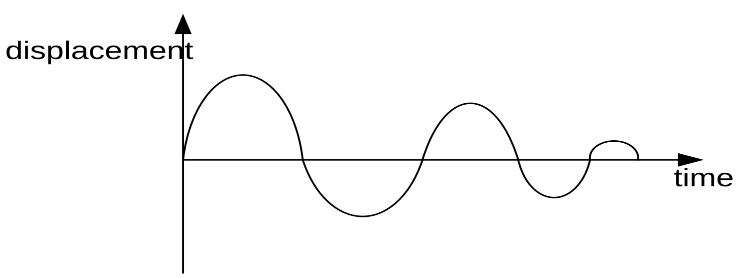
b) Damped oscillations

These are oscillations in which energy is lost and amplitude keeps on decreasing until it dies away due to dissipative forces.

Types of damped oscillations

i) Under damped/slightly damped/lightly damped oscillations

Is when energy is lost and amplitude gradually decreases until oscillation dies away.

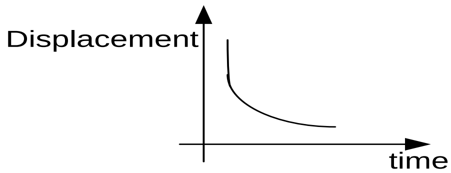


Examples

- Mass oscillating at the end of the spring oscillating in air
- Simple pendulum oscillating in air

ii)Over damped/highly damped/heavily damped

Is when a system does not oscillate when displaced but takes a very long time to return to equilibrium position.

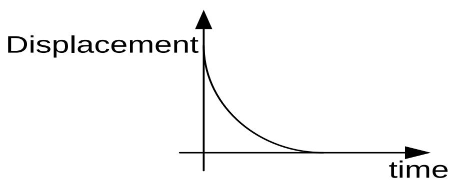


Example

- ❖ A horizontal spring with a mass on a rough surface

iii) Critically damped oscillations

Is when a system does not oscillate when displaced and returns to equilibrium position in a short time.



Example

- ❖ Shock absorber in a car

C) FORCED OSCILLATIONS

These are vibrations caused by an external force and the system oscillates at the same frequency as the vibrating force.

Example

- ❖ Oscillation of a guitar string
- ❖ Oscillation of a building during an earthquake
- ❖ Oscillation of air column in a musical pipe

UNEB 2013 No4

(b) Explain **Brownian motion** (03marks)

(c) Explain the energy changes which occur when a pendulum is set into motion (03marks)

An[p.e to k.e to p.e]

(d) A simple pendulum of length 1 m has a bob of mass 100g. It is displaced from its mean Position A to a position B so that the string makes an angle of 45° with the vertical. Calculate the ;

(i) Maximum potential energy of the bob (03marks)

(ii) Velocity of the bob when the string makes angle of 30° with the vertical. [Neglect air resistance] (04marks)

Solution

i) $P.e = mgh$

$= mg(l - lCos\theta)$

$= 0.1x9.81(1 - 1cos45)$

$P.e = 0.287J$

ii) By law of conservation of energy

$K.e = P.e$

$\frac{1}{2}mv^2 = mgh$

$\frac{1}{2}mv^2 = mg(l - lCos\theta)$

$\boldsymbol{v} = \sqrt{2g(l - lCos\theta)}$

$\boldsymbol{v} = \sqrt{2x9.81(1 - 1Cos30)}$

$v = 0.164ms^{-1}$

UNEB 2012 No 2

a) Define the following terms as applied to oscillating motion

i) Amplitude [1mk]

ii) Period [1mk]

b) State four characteristics of simple harmonic motion [2mk]

c) A mass m, is suspended from a rigid support by a string of length, l. the mass is pulled a side so that the string makes an angle, θ with the vertical and then released.

i) Show that the mass executes simple harmonic motion with a period, $T = 2\pi\sqrt{\frac{l}{g}}$ [05mk]

ii) Explain why this mass comes to a stop [02mk]

d) A piston in a car engine performs simple harmonic motion of frequency 12.5Hz. If the mass of the piston is 0.50kg and its amplitude of vibration is 45mm, find the maximum force on the piston. **An[139N]** [03mk]

e) Describe an experiment to determine the acceleration due to gravity, g using a spiral spring of known force constant [06mk]

UNEB 2011 No 2

a) i) what is meant by simple harmonic motion [1mk]

ii) State two practical examples of simple harmonic motion [1mk]

iii) Using graphical illustration distinguish between under damped and critically damped oscillation [4mk]

b) i)describe an experiment to measure acceleration due to gravity using a spiral spring [6mk]

ii) State two limitations to the accuracy of the value it b (i) [02mk]

UNEB 2010 No 2

b) i) What is meant by a simple harmonic motion [1mk]

ii) Distinguish between damped and forced oscillations [2mk]

c) a cylinder of length l, cross sectional area A and density,δ, floats in a liquid of density, ρ, the cylinder is pushed down slightly and released.

i) Show that a performs simple harmonic oscillation [5mk]

ii) Derive the expression for the period of oscillation [2mk]

An($T=2\pi\sqrt{\left(\frac{\delta\, l}{\rho\, g}\right)}$ **)**

d) A spring of force constant $40Nm^{-1}$ is suspended vertically. A man of 0.1kg suspended from the spring is pulled down a distance of 5mm and released. Find the,

i) Period of oscillation **An[0.314s]** [2mk]

ii) Maximum oscillation of the mass **An[2ms^{-2}]** [2mk]

iii) Net force acting on the mass when it is 2mm below the centre of oscillation. **An[0.08N]** [2mk]

UNEB 2009 No 3

(a) What is meant by simple harmonic motion (01marks)

(b) A cylindrical vessel of cross-sectional area A, contains air of volume V, at a pressure P, trapped by frictionless air tight piston of mass M,. The piston is pushed down and released.

(i) If the piston oscillates with s.h.m, show that the frequency is given by $f = \frac{A}{2\pi}\sqrt{\frac{P}{m\,v}}$ (06marks)

(ii) Show that the expression for, f in b(i) is dimensionally correct (02marks)

(c) Particle executing s.h.m vibrates in a straight line, given that the speeds of the particle are $4ms^{-1}$and $2ms^{-1}$ when the particle is 3cm and 6cm respectively from equilibrium. calculate the;

(i) amplitude of oscillation **An(6.7x10^{-2}m)** (03marks)

(ii) frequency of the particle **An(10.68Hz)** (03marks)

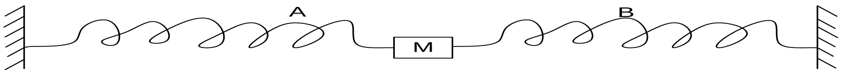
(d) Give two examples of oscillatory motions which execute s.h.m and state the assumptions made in each case

UNEB 2008 No3

a) (i) Define simple harmonic motion [01marks]

(ii) A particle of mass m executes simple harmonic between two point A and B about equilibrium position O. Sketch a graph of the restoring force acting on the particle as a function of distance r and moved by the particle [02marks]

b)



Two springs A and B of spring constants K_A and K_B respectively are connected to a mass m as shown. The surface on which the mass slides is frictionless.

(i) Show that when the mass is displaced slightly, it oscillates with simple harmonic motion of frequency given by

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{k_A + k_B}{m}\right)}$$ [04marks]

(ii) If the two springs above are identical such that $k_A = k_B = 5\text{Nm}^{-1}$ and mass m=50g, calculate the period of oscillation **An[0.44s]** [03marks]

UNEB 2007 No 1

a) Define simple harmonic motion [01marks]

b) Sketch a graph of

i) velocity against displacement [03marks]

ii) acceleration against displacement for a body executing S.H.M

c) A glass U-tube containing a liquid is tilted slightly and then released

i) Show that the liquid oscillates with S.H.M [04marks]

ii) Explain why the oscillations ultimately come to rest [03marks]

UNEB 2007 No 4

b) i)What is meant by acceleration due to gravity [01mark]

ii)Describe how you would use a spiral string, a retort stand with a clamp, a pointer, seven 50g masses, meter rule and a stop clock to determine the acceleration due to gravity [6mk]

iii) State any two sources of errors in the experiment in bii) above. [01mark]

iv)A body of mass 1kg moving with simple harmonic motion has speed of 5ms^{-1} and 3ms^{-1} when it is at a distance of 0.1m and 0.2m respectively from the equilibrium point. Find the amplitude of motion [04marsk]