

S6 SPECIAL APPLIED MATHEMATICS TEST 1 JUNE 2020

TIME: 2HOURS 30 MINUTES

Answer **all** the questions and cross check with the corresponding answers.

1. Two events A and B are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{3}{8}$ and $P\left(\frac{A}{B}\right) = \frac{7}{12}$. Find
(i) $P(A \cap B')$ (ii) $P\left(\frac{B}{A'}\right)$ (Ans: $\frac{9}{32}, \frac{5}{16}$)

2. A particle moving with a uniform acceleration has a velocity of $(6 + 3t)\mathbf{i} - (1 + 2t)\mathbf{j} \text{ ms}^{-1}$ at any time t . Determine the time when the speed is 13 ms^{-1} . (Ans: 2 s)

3. The masses of packets of tea leaves from a certain factory are normally distributed with a mean 6.5g and standard deviation 0.024g. Find the probability that a packet selected at random has a mass between 6.48g and 6.53g. (Ans: 0.6919)

4. The table below shows an extract for values of $\sin x^\circ$;

$x = 10^\circ$	18'	24'	30'
$\sin x$	0.1788	0.1805	0.1822

Use linear interpolation or extrapolation to estimate

- (i) $\sin 10^\circ 36'$ (ii) $\sin^{-1} 0.1792$ (Ans: 0.1839, $10^\circ 19.4'$)
5. A body initially at a point (40, 10, 20) is moving with a uniform speed of 5 ms^{-1} in a direction $4\mathbf{i} + 7\mathbf{j} + 4\mathbf{k}$. Calculate its distance from the origin after 9 seconds. (Ans: 85 m)
6. Show that the equation $\ln(x+2) - 2x - 1 = 0$ has a root between -1 and 0. Hence use linear interpolation once to find a better root to the equation. (Ans: -0.2)
7. A particle of mass 8 kg is attached to one end of a light inextensible string of length 50 cm fastened at a point A. The particle is held in equilibrium by a horizontal force P, at a point, 30 cm from a vertical through A. Show that the magnitude of P is 58.8 N.
8. Seven students were given two separate aptitude tests and their grades are shown in the table below.

Test 1	F	O	A	D	B	C	E
Test 2	D	O	E	B	F	C	A

Calculate a rank correlation coefficient for the above information and comment on your result. (Ans: -0.25)

9. The table below shows the ages of patients recorded in a hospital on a certain day.

Ages(yrs)	1 - 10	11 - 20	21 - 30	31 - 40	41 - 50
No. of patients	6	12	30	28	4

Calculate the median age and standard deviation. (Ans: 28, 9.8869)

10. A discrete random variable X has a probability distribution defined by

$$P(X = x) = \begin{cases} \frac{k}{x} & x = 1, 2, 3, 4 \\ 0 & \text{elsewhere} \end{cases} \text{ . Find (i) the value of the constant } k$$

$$(ii) \quad \text{Var}(3X) \quad (iii) \quad P(|X - 2| \leq 1) \quad (\text{Ans: } \frac{12}{25}, 10.0224, \frac{22}{25})$$

11. Use the trapezium rule with six ordinates to estimate $\int_0^{\frac{\pi}{3}} e^x \cos x \, dx$ correct to three decimal places. (Ans: 1.439)

12. A particle of mass 4 kg is acted upon by a time varying force given by

$$\mathbf{F} = \begin{pmatrix} 2 + 3t + 4e^t \\ 1 \\ 2t \end{pmatrix} N \text{ . Initially the particle has a velocity } \mathbf{i} + 4\mathbf{j} + 2\mathbf{k} \text{ at the}$$

point with position vector $4\mathbf{i} - \mathbf{j} + 3\mathbf{k}$. Show that the displacement of the

$$\text{particle after time } t \text{ is given by } \begin{pmatrix} \frac{t^2}{4} + \frac{t^3}{8} + e^t + 3 \\ 4t + \frac{t^2}{8} - 1 \\ \frac{t^3}{12} + 2t + 3 \end{pmatrix} m \text{ .}$$