P425/2
PURE
MATHEMATICS
PAPER 1
August. 2017
3 hour

NDEJJE SENIOR SECONDARY SCHOOL
Uganda Advanced Certificate of Education
MOCK SET 4 EXAMINATIONS 2017
PURE MATHEMATICS
Paper 1
3 hours

INSRUCTIONS TO CANDIDATES:

Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.

Additional question(s) answered will **not** be marked.

All working **must** be shown clearly.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 MARKS)

(Answer **all** questions in this section.)

Qn 1: The sum of the second and third terms of a Geometric Progrssion (G.P) is 48. The sum of the fifth and sixth terms is 1296. Find the common ratio, the first term and the sum of the first 12 terms of the G.P. [5]

Qn 2: Use De Moivre's theorem to prove that
$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$. [5]

Qn 3: When a polynomial $P(x)$ is divided by $x^2 - 5x - 14$, the remainder is $2x + 5$. Find the remainder when $P(x)$ is divided by:
(i). $x - 7$,
(ii). $x + 2$. [5]

Qn 4: OAB is a triangle in which $= \underset{\sim}{\boldsymbol{a}}$, $OB = \underset{\sim}{\boldsymbol{b}}$. C is a point on AB such that $AC: CB = 3:1$. D is the midpoint of OA. DC and OB, both produced meet in point T. Find vector OT in terms of $\underset{\sim}{\boldsymbol{a}}$ and $\underset{\sim}{\boldsymbol{b}}$. [5]

Qn 5: Find the integral $\int x\cos^2 x\, dx$. [5]

Qn 6: Given that $y = x + a$ is a tangent to the curve $y = ax^2 + bx + c$ at the point $(2, 4)$. Find the values of the constants a, b and c. [5]

Qn 7: Find the volume of the solid of revolution generated when the area under $y = \frac{1}{x-2}$ from $x = 3$ to $x = 4$ is rotated through four right angles about the x-axis. [5]

Qn 8: In triangle ABC, $AB = x - y$, $BC = x + y$ and $CA = x$, show that $\cos A = \frac{x-4y}{2(x-y)}$. [5]

SECTION B (60 MARKS)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

Question 9:

(a). Find the centroid of the triangle whose sides are given by the equations $x + y = 11, y = x - 1$ and $3y = x - 3$. [5]

(b). ABCD is a rhombus such that the coordinates $A(-3, -4)$ and $C(5, 4)$. Find the equation of the diagonal BD of the rhombus. If the gradient of side BC is 2, obtain the coordinates of B and D, prove that the area of the rhombus is $21\frac{1}{3}$ square units. [7]

Question 10:

Show that $\int_0^1 \frac{x^2+6}{(x^2+4)(x^2+9)} dx = \frac{\pi}{20}$. [12]

Question 11:

(a). Using Maclaurin's theorem, expand $e^{-x} \sin x$ upto the term in x^3. Hence evaluate $e^{-\frac{\pi}{3}} \sin \frac{\pi}{3}$ to four significant figures. [5]

(b). The curve $y = x^3 + 8$ cuts the x and y axes at the points A and B respectively. The line AB meets the curve again at point C. Find the coordinates of A, B and C hence find the area enclosed between the curve and the line. [7]

Question 12:

(a). The position vectors of the points P and Q are $4\underset{\sim}{\boldsymbol{i}} - 3\underset{\sim}{\boldsymbol{j}} + 5\underset{\sim}{\boldsymbol{k}}$ and $\underset{\sim}{\boldsymbol{i}} + 2\underset{\sim}{\boldsymbol{j}}$ respectively. Find the coordinates of the point R such that $PQ: PR = 2: 1$. [4]

(b). If the vector $5\underset{\sim}{\boldsymbol{i}} - \lambda\underset{\sim}{\boldsymbol{j}} + \underset{\sim}{\boldsymbol{k}}$ is perpendicular to the line $\underset{\sim}{\boldsymbol{r}} = \underset{\sim}{\boldsymbol{i}} - 4\underset{\sim}{\boldsymbol{j}} + t(2\underset{\sim}{\boldsymbol{i}} + 3\underset{\sim}{\boldsymbol{j}} - 4\underset{\sim}{\boldsymbol{k}})$**.** Find the value of λ. [3]

(c). Obtain the equation of the plane that passes through $(1, -2, 2)$ and it's perpendicular to the line $\frac{x-9}{4} = \frac{y-6}{-1} = \frac{z-8}{1}$. [5]

Question 13:

The parametric equations $x = \frac{1+t}{1-t}$ and $y = \frac{2t^2}{1-t}$ represent a curve.

(i). Find the cartesian equation of the curve. [4]

(ii). Determine the turning points of the curve and their nature. [3]

(iii). State the asymptotes and intercepts of the curve. [3]

(iv). Hence sketch the curve. [2]

Question 14:

(a). Determine the maximum value of the expression $6\sin x - 3\cos x$. [3]

(b). Prove that $\frac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°} = \tan 56°$. [3]

(c). In a triangle ABC, prove that $\sin B + \sin C - \sin A = 4\cos\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$. [6]

Question 15:

(a). Simplify $(2+5i)^2 + 5\frac{(7+2i)}{3-4i} - i(4-6i)$ expressing your answer in the form $a + bi$. [5]

(b). If $z = x + yi$, where $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$. Show that the locus of $Arg\left(\frac{z-1}{z-i}\right) = \frac{\pi}{3}$ is a circle. Find its centre and radius. [7]

Question 16:

(a). Using the substitution $y = ux$, solve the differential equation $x^2\frac{dy}{dx} = x^2 + xy + y^2$. [4]

(b). The rate at which a liquid runs from a container is proportional to the square root of the depth of the opening below the surface of the liquid. A cylindrical petrol storage tank is sunk in the ground with its axis vertical. There is a leak in the tank at an unknown depth. The level of the petrol in the tank originally full is found to drop by 20 cm in 1 hour and by 19 cm in the next hour. Find the depth at which the leak is located. [8]

*****END*****

NDEJJE SENIOR SECONDARY SCHOOL

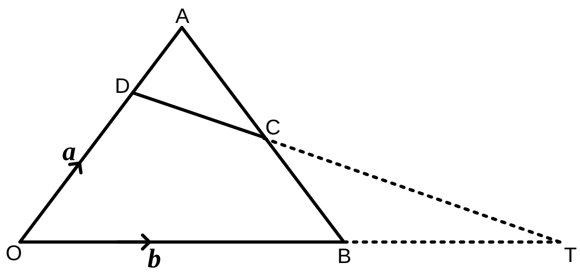
Uganda Advanced Certificate of Education

MARKING GUIDE FOR MOCK SET 4 EXAMINATIONS 2017

PURE MATHEMATICS

Paper 1

SNo.	Working	Marks
1	$ar + ar^2 = 48, \quad \Rightarrow ar(1+r) = 48 \quad \rightarrow (1)$ $ar^4 + ar^5 = 1296, \quad \Rightarrow ar^4(1+r) = 1296 \quad \rightarrow (2)$ $(2) \div (1)$ gives; $\frac{ar^4(1+r)}{ar(1+r)} = \frac{1296}{48}\ , \quad \Rightarrow r^3 = 27, \quad \Rightarrow r = 3$ From (1); $a = \frac{48}{r(1+r)} = \frac{48}{3(1+3)} = 4$ $\Rightarrow S_{12} = 4\left(\frac{3^{12}-1}{3-1}\right) = 1062880$	**B1** –for eqns 1 & 2 **M1** -solving **A1** –for r. **M1** **A1**
2	$(\cos\theta + i\sin\theta)^5 = \cos 5\theta + i\sin 5\theta$ $\cos^5\theta + 5i\cos^4\theta\sin\theta - 10\cos^3\theta\sin^2\theta - 10i\cos^2\theta\sin^3\theta$ $+ 5\cos\theta\sin^4\theta + i\sin^5\theta = \cos 5\theta + i\sin 5\theta$ By comparison, $\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$ $= \cos^5\theta - 10\cos^3\theta\,(1-\cos^2\theta) + 5\cos\theta\,(1-\cos^2\theta)^2$ $= \cos^5\theta - 10\cos^3\theta + 10\cos^5\theta$ $+ 5\cos\theta\,(1 - 2\cos^2\theta + \cos^4\theta)$ $= -10\cos^3\theta + 11\cos^5\theta + 5\cos\theta - 10\cos^3\theta + 5\cos^5\theta$ $= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$	**B1** -equating **M1** - expanding **M1** –equating real parts **M1** - simplification **A1**
3	(i). $g(x) = x^2 - 5x - 14 = (x-7)(x+2)$ let, $\quad R(x) = 2x + 5$ for, $\quad (x-7) = 0, x = 7, \quad \Rightarrow R(7) = 2(7) + 5 = 19$ (ii). for, $\quad (x+2) = 0, x = -2, \quad \Rightarrow R(-2) = 2(-2) + 5 = 1$	 **B1** **M1 A1** **M1 A1**

<table>
<tr>
<td>4</td>
<td>


$$OD = DA = \frac{1}{2}\underset{\sim}{\boldsymbol{a}}, \qquad AB = OB - OA = \underset{\sim}{\boldsymbol{b}} - \underset{\sim}{\boldsymbol{a}}$$
$$AC: CB = 3:1, \qquad \Longrightarrow AC = \frac{3}{4}AB = \frac{3}{4}\underset{\sim}{\boldsymbol{b}} - \frac{3}{4}\underset{\sim}{\boldsymbol{a}}$$
$$DT = \mu DC = \mu(DA + AC) = \mu\left[\frac{1}{2}\underset{\sim}{\boldsymbol{a}} + \frac{3}{4}\underset{\sim}{\boldsymbol{b}} - \frac{3}{4}\underset{\sim}{\boldsymbol{a}}\right] = \frac{3}{4}\mu\underset{\sim}{\boldsymbol{b}} - \frac{1}{4}\mu\underset{\sim}{\boldsymbol{a}}$$
$$OT = \lambda OB = \lambda\underset{\sim}{\boldsymbol{b}}$$
$$OT = OD + DT$$
$$\lambda\underset{\sim}{\boldsymbol{b}} = \frac{1}{2}\underset{\sim}{\boldsymbol{a}} + \frac{3}{4}\mu\underset{\sim}{\boldsymbol{b}} - \frac{1}{4}\mu\underset{\sim}{\boldsymbol{a}}$$
Comparing coefficients of $\underset{\sim}{\boldsymbol{a}}$ gives:
$$0 = \frac{1}{2} - \frac{1}{4}\mu, \qquad \Longrightarrow \mu = 2$$
Comparing coefficients of $\underset{\sim}{\boldsymbol{b}}$ gives:
$$\lambda = \frac{3}{4}\mu = \frac{3}{4}\times 2 = \frac{3}{2}, \qquad \Longrightarrow OT = \lambda\underset{\sim}{\boldsymbol{b}} = \frac{3}{2}\underset{\sim}{\boldsymbol{b}}$$
</td>
<td>
B1 –vector diagram

B1 –for AC
B1 –for DT

B1 –for μ

B1 –for OT
</td>
</tr>
<tr>
<td>5</td>
<td>

$$\int x\cos^2 x\,dx = \int x\left(\frac{1+\cos 2x}{2}\right)dx$$
$$= \frac{1}{2}\int x\,dx + \frac{1}{2}\int x\cos 2x\,dx$$

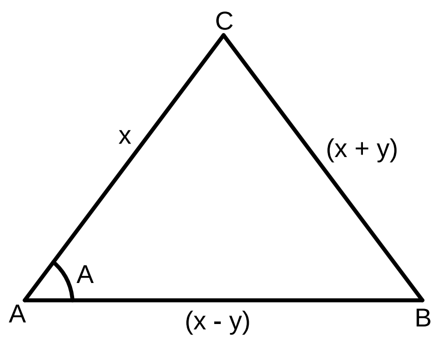
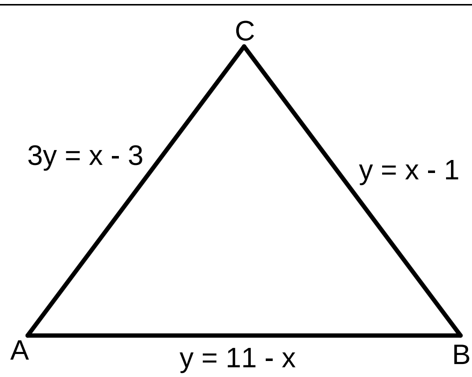
Sign	Differentiation	Integration
$+$	x	$\cos 2x$
$-$	1	$\frac{1}{2}\sin 2x$
$+$	0	$-\frac{1}{4}\cos 2x$

$$\int x\cos^2 x\,dx = \frac{1}{2}x^2 + \frac{1}{2}\left[\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x\right] + c$$
$$= \frac{1}{2}x^2 + \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x + c$$
</td>
<td>
B1

B1
B1
B1

M1 M1 – substitution & simplification
A1
</td>
</tr>
<tr>
<td>6</td>
<td>

$$y = ax^2 + bx + c, \qquad \Longrightarrow \frac{dy}{dx} = 2ax + b$$
At point $(2, 4)$,
</td>
<td>B1 – for dy/dx</td>
</tr>
</table>

	$y = x + a, \quad \Rightarrow 4 = 2 + a, \quad \Rightarrow a = 2$ gradient, $\frac{dy}{dx} = 2 \times 2 \times 2 + b = 1, \quad \Rightarrow b = -7$ $y = ax^2 + bx + c, \quad \Rightarrow 4 = 2(2)^2 + (-7)(2) + c$ $\Rightarrow 4 = 8 - 14 + c, \quad \Rightarrow c = 10$ $\therefore a = 2, \quad b = -7, \quad c = 10$	**M1** –solving **A1** –for a **A1** –for b **A1** –for c
7	$\text{Volume} = \pi \int_3^4 y^2\,dx = \pi \int_3^4 (x-2)^{-2}\,dx = \pi \left[\frac{(x-2)^{-1}}{-1}\right]_3^4$ $= \pi \left[\frac{1}{2-x}\right]_3^4 = \pi\left(-\frac{1}{2} + 1\right) = \frac{1}{2}\pi$ cubic units	**M1 M1** **M1 M1 A1**
8	 By cosine rule, $(x+y)^2 = x^2 + (x-y)^2 - 2x(x-y)\cos A$ $x^2 + 2xy + y^2 = x^2 + x^2 - 2xy + y^2 - 2x(x-y)\cos A$ $4xy - x^2 = -2x(x-y)\cos A$ $x - 4y = 2(x-y)\cos A$ $\cos A = \frac{x-4y}{2(x-y)}$	**B1** **M1** – substitution **M1 M1** – simplification **A1**
9	 At point A, $3(x - 11) = x - 3, \quad \Rightarrow x = 15$ when, $x = 15, \quad y = 15 - 11 = 4, \quad \Rightarrow A(15, 4)$ At point B, $11 - x = x - 1, \quad \Rightarrow x = 6$	**B1** **M1 B1**

$$\text{when,} \quad x = 6, \quad y = 6 - 1 = 5, \quad \Rightarrow B(6, 5)$$

At point C,

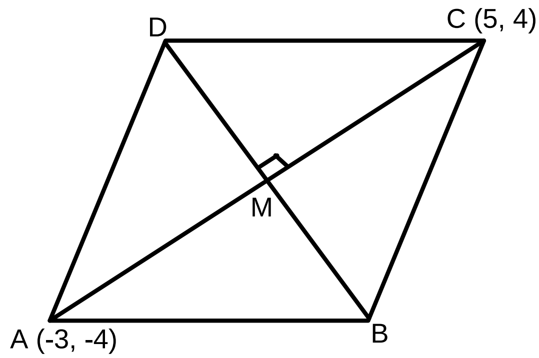
$$3(x-1) = x - 3, \quad \Rightarrow x = 0$$

$$\text{when,} \quad x = 0, \quad y = 0 - 1 = -1, \quad \Rightarrow C(0, -1)$$

$$\text{centroid} = \left(\frac{15+6+0}{3}, \frac{4+5-1}{3}\right) = \left(7, \frac{8}{3}\right)$$

M1 A1

(b).



$$\text{Gradient of AC} = \frac{-4-4}{-3-5} = 1, \quad \Rightarrow \text{Gradient of BD} = -1$$

B1 –graident AC

$$\text{Midpoint of AC,} \quad M\left(\frac{-3+5}{2}, \frac{-4+4}{2}\right) = (1, 0)$$

The equation of line BD is given by:

$$\frac{y-0}{x-1} = -1, \quad \Rightarrow y = -x + 1$$

B1 –equation BD

The equation of line BC is given by:

$$\frac{y-4}{x-5} = 2, \quad \Rightarrow y = 2x - 6$$

B1 –equation BC

At point B,

$$-x + 1 = 2x - 6, \quad \Rightarrow x = \frac{7}{3}$$

$$x = \frac{7}{3}, \quad y = -\frac{7}{3} + 1 = -\frac{4}{3}, \quad \Rightarrow B\left(\frac{7}{3}, -\frac{4}{3}\right)$$

$$\text{Midpoint of AC} = \left(\frac{\frac{7}{3}+x}{2}, \frac{-\frac{4}{3}+y}{2}\right) = (1, 0)$$

B1

$$\frac{7}{3} + x = 2, \quad \Rightarrow x = \frac{1}{3}$$

$$-\frac{4}{3} + y = 0, \quad \Rightarrow y = \frac{4}{3}$$

$$\Rightarrow D\left(\frac{1}{3}, \frac{4}{3}\right)$$

B1 –for D

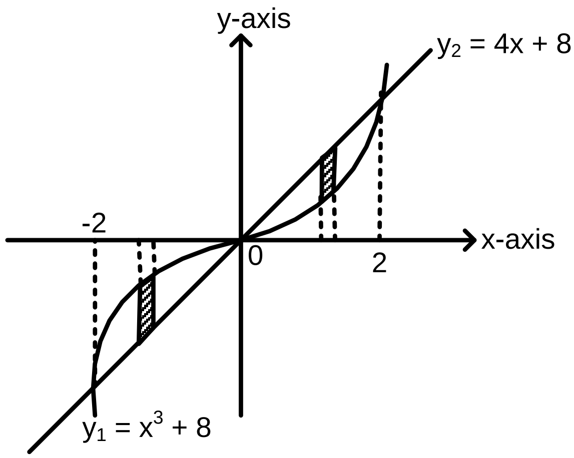
The coordinates of B and D are $B\left(\frac{7}{3}, -\frac{4}{3}\right)$ and $D\left(\frac{1}{3}, \frac{4}{3}\right)$.

$$AC = OC - OA = \begin{pmatrix}5\\4\end{pmatrix} - \begin{pmatrix}-3\\-4\end{pmatrix} = \begin{pmatrix}8\\8\end{pmatrix}$$

	$MB = OB - OM = \frac{1}{3}\begin{pmatrix}7\\-4\end{pmatrix} - \begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{3}\begin{pmatrix}4\\-4\end{pmatrix}$ $\text{Area} = \lvert AC\rvert\lvert MB\rvert = \sqrt{8^2+8^2} \times \frac{1}{3}\sqrt{4^2+4^2} = \frac{64}{3}$ $= 21\frac{1}{3}$ sq. units	**M1** **A1**
10	$\frac{x^2+6}{(x^2+4)(x^2+9)} \equiv \frac{Ax+B}{(x^2+4)} + \frac{Cx+D}{(x^2+9)}$ $x^2+6 \equiv (Ax+B)(x^2+9) + (Cx+D)(x^2+4)$ Comparing coefficients of; x^0, $9B+4D=6 \longrightarrow (1a)$ x^1, $9A+4C=0 \longrightarrow (1b)$ x^2, $B+D=1 \longrightarrow (1c)$ x^3, $A+C=0 \longrightarrow (1d)$ Equation $(1a)-(1c)$ gives: $5B=2, \quad \Longrightarrow B=\frac{2}{5}$ From equation $(1c)$; $D = 1-B = 1-\frac{2}{5} = \frac{3}{5}$ Equation $(1b)-(1d)$ gives: $5A=0, \quad \Longrightarrow A=0$ From equation $(1c)$; $C=-A=0$ $\frac{x^2+6}{(x^2+4)(x^2+9)} \equiv \frac{2}{5(x^2+4)} + \frac{3}{5(x^2+9)}$ $\int_0^1 \frac{x^2+6}{(x^2+4)(x^2+9)}dx = \frac{2}{5}\int_0^1 \frac{1}{(x^2+4)}dx + \frac{3}{5}\int_0^1 \frac{1}{(x^2+9)}dx$ $= \frac{2}{5}\left[\tan^{-1}\left(\frac{x}{2}\right)\right]_0^1 + \frac{3}{5}\left[\tan^{-1}\left(\frac{x}{3}\right)\right]_0^1$ $= \frac{2}{5}\left[\frac{1}{2}\tan^{-1}\left(\frac{1}{2}\right) - 0\right] + \frac{3}{5}\left[\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\right) - 0\right]$ $= \frac{1}{5}\tan^{-1}\left(\frac{1}{2}\right) + \frac{1}{5}\tan^{-1}\left(\frac{1}{3}\right) = \frac{1}{5}\left[\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right)\right]$ let, $\alpha = \tan^{-1}\left(\frac{1}{2}\right), \quad \Longrightarrow \tan\alpha = \frac{1}{2}$ let, $\beta = \tan^{-1}\left(\frac{1}{2}\right), \quad \Longrightarrow \tan\beta = \frac{1}{3}$ $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} = \left(\frac{1}{2}+\frac{1}{3}\right)\Big/\left(1-\frac{1}{2}\times\frac{1}{3}\right) = \frac{5}{6} \div \frac{5}{6}$ $= 1$ $\Longrightarrow (\alpha+\beta) = \tan^{-1}1 = \frac{\pi}{4}$	**M1** **M1** **M1** **A1** **A1** **A1** **B1** **M1** **M1** **M1** **M1**

	$\int_0^1 \frac{x^2+6}{(x^2+4)(x^2+9)}dx = \frac{1}{5}\left[\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{3}\right)\right] = \frac{1}{5}(\alpha+\beta)$ $= \frac{1}{5}\times\frac{\pi}{4} = \frac{\pi}{20}$	**B1**
11	(a). Let, $f(x) = e^{-x}\sin x, \quad \Longrightarrow f(0) = e^0\sin 0 = 0$ $f'(x) = e^{-x}\cos x - e^{-x}\sin x = e^{-x}(\cos x - \sin x),$ $\Longrightarrow f'(0) = 1$ $f''(x) = e^{-x}(-\sin x - \cos x) - e^{-x}(\cos x - \sin x)$ $= -2e^{-x}\cos x, \quad \Longrightarrow f''(0) = -2$ $f'''(x) = 2e^{-x}\sin x + 2e^{-x}\cos x = 2e^{-x}(\sin x + \cos x),$ $\Longrightarrow f'''(0) = 2$ By Maclaurin's theorem, $f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \cdots$ $= 0 + x\times 1 + \frac{x^2}{2!}\times(-2) + \frac{x^3}{3!}\times 2 + \cdots$ $\therefore e^{x}\sin x = x - x^2 + \frac{1}{3}x^3 + \cdots$ For the hence part, $e^{-\frac{\pi}{3}}\sin\frac{\pi}{3} = \frac{\pi}{3} - \left(\frac{\pi}{3}\right)^2 + \frac{2}{3}\left(\frac{\pi}{3}\right)^3 \approx 0.3334 \text{ (4 s.f)}$ (b). $y = x^3 + 8$ when, $y = 0, \quad 0 = x^3 + 8, \quad x = -2, \quad \Longrightarrow A(-2, 0)$ when, $x = 0, \quad y = 0 + 8 = 8, \quad \Longrightarrow B(0, 8)$ The equation of line AB is given by: $\frac{y-8}{x-0} = \frac{0-8}{-2-0}, \quad \Longrightarrow y = 4x + 8$ When the line AB meets the curve, $4x + 8 = x^3 + 8, \quad \Longrightarrow x(4 - x^2) = 0$ $x = 0, \quad \text{or}, \quad x = \pm 2$ when, $x = 2, \quad y = 8 + 8 = 16, \quad \Longrightarrow C(2, 16)$ (ii).	 **B1** **B1** **M1** **A1** **M1 A1** **B1** –for A & B **B1**

<table>
<tr>
<td></td>
<td>



$$\text{Area} = \int_{-2}^{0} (y_1 - y_2)\,dx + \left|\int_{0}^{2} (y_2 - y_1)\,dx\right|$$

$$\int_{-2}^{0} (y_1 - y_2)\,dx = \int_{-2}^{0} (x^3 - 4x)\,dx = \left[\frac{1}{4}x^4 - 2x^2\right]_{-2}^{0}$$

$$= 0 - (4 - 8) = 4$$

$$\int_{0}^{2} (y_2 - y_1)\,dx = \int_{0}^{2} (4x - x^3)\,dx = \left[2x^2 - \frac{1}{4}x^4\right]_{0}^{2}$$

$$= (4 - 8) - 0 = -4$$

$$\text{Area} = \int_{-2}^{0} (y_1 - y_2)\,dx + \left|\int_{0}^{2} (y_2 - y_1)\,dx\right| = 4 + |-4|$$

$$= 8\ \text{sq. units}$$

</td>
<td>

B1

M1

A1

</td>
</tr>
<tr>
<td>12</td>
<td>

(a).

$$PQ = OQ - OP = \begin{pmatrix}1\\2\\0\end{pmatrix} - \begin{pmatrix}4\\-3\\5\end{pmatrix} = \begin{pmatrix}-3\\5\\5\end{pmatrix}$$

$$PQ:PR = 2:1, \qquad \Rightarrow \frac{PQ}{PR} = \frac{2}{1}, \qquad \Rightarrow PR = \frac{1}{2}PQ$$

$$OR = OP + \frac{1}{2}PQ = \begin{pmatrix}4\\-3\\5\end{pmatrix} + \frac{1}{2}\begin{pmatrix}-3\\5\\5\end{pmatrix} = \begin{pmatrix}2.5\\-0.5\\7.5\end{pmatrix}$$

The coordinates are $R(2.5, -0.5, 7.5)$.

(b). For perpendicular vectors,

$$\begin{pmatrix}5\\-\lambda\\1\end{pmatrix}.\begin{pmatrix}2\\3\\-4\end{pmatrix} = 0$$

$$10 - 3\lambda - 4 = 0, \qquad \Rightarrow \lambda = 2$$

(c).

</td>
<td>

B1 –for PQ

B1 –for PR

B1 –for OR

B1 –for coordinate R

M1 M1 – dotting and equating to zero.

B1 –for λ

</td>
</tr>
</table>

	Normal vector, $\underset{\sim}{\boldsymbol{n}} = \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}$ Position vector, $\underset{\sim}{\boldsymbol{a}} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ $\underset{\sim}{\boldsymbol{r}}.\underset{\sim}{\boldsymbol{n}} = \underset{\sim}{\boldsymbol{n}}.\underset{\sim}{\boldsymbol{a}}$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix}.\begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}.\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ $4x - y + z = 4 + 2 + 2$ $4x - y + z = 8$	**B1** **B1** **B1** **M1** **A1**
13	(i). From, $x = \dfrac{1+t}{1-t}$, $x - tx = 1 + t,$ $\Rightarrow t = \dfrac{x-1}{x+1}$ $t^2 = \dfrac{x^2 - 2x + 1}{x^2 + 2x + 1}$ $y = \dfrac{2t^2}{1-t} = \dfrac{2\left(\frac{x-1}{x+1}\right)^2}{\left(1 - \frac{x-1}{x+1}\right)} = \dfrac{2\left(\frac{x-1}{x+1}\right)^2}{\left(\frac{2}{x+1}\right)} = \dfrac{(x-1)^2}{(x+1)}$ $\Rightarrow y = \dfrac{(x-1)^2}{(x+1)}$	**B1** –for t **M1** – substitution **A1**
	(ii). $\dfrac{dy}{dx} = \dfrac{(x+1) \times 2(x-1) - (x-1)^2}{(x+1)^2}$ For turning points, $\frac{dy}{dx} = 0$ $\dfrac{2(x+1)(x-1) - (x-1)^2}{(x+1)^2} = 0$ $(x-1)[2(x+1) - (x-1)] = 0$ $(x-1)(x+3) = 0$ $x = 1,$ or, $x = -3$ when, $x = 1,$ $y = \dfrac{(1-1)^2}{(1+1)} = 0$ when, $x = -3,$ $y = \dfrac{(-3-1)^2}{(-3+1)} = -8$ The turning points are: $(1, 0)$ and $(-3, -8)$.	**M1** **A1** **B1** –turning points

x	L	-3	R		L	1	R
Sign of $\frac{dy}{dx}$	$+$	0	$-$		$-$	0	$+$
Nature		Max				Min	

(iii).

$$y = \frac{(x-1)^2}{(x+1)} = \frac{x^2 - 2x + 1}{x + 1}$$

By synthetic method

	1	-2	1
$x = -1$		-1	3
	1	-3	4

$y = x - 3 + \dfrac{4}{x+1}$, $\quad \Longrightarrow y = x - 3$ is the slanting asymptote — **B1** –slanting asymptote

Vertical asymptote

$$as\ y \to \infty, (x+1) \to 0$$

$\Longrightarrow x = -1$ is the vetical asymptote — **B1** –vertical asymptote

Intercepts

$$y = \frac{(x-1)^2}{(x+1)}$$

when, $\quad x = 0, \quad y = 1$

when, $\quad y = 0, \quad (x-1)^2 = 0, \quad \Longrightarrow x = 1$

The intercepts are $(0, 1)$ and $(1, 0)$. — **B1** – intercepts

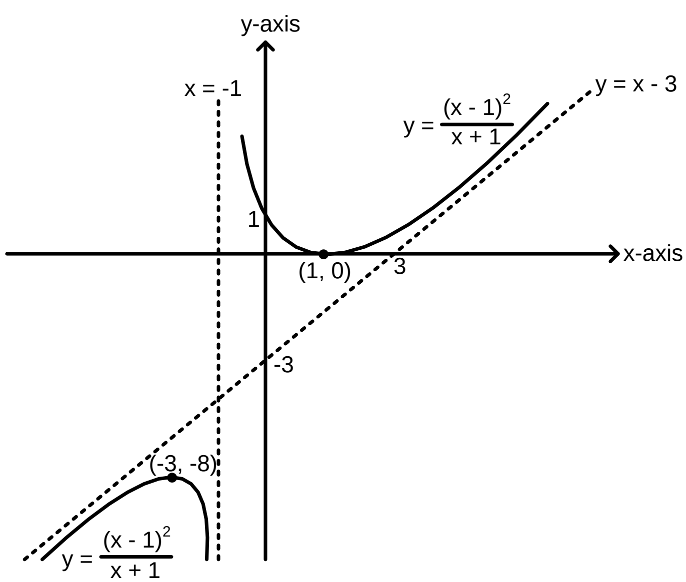
(iv). The Critical values include: $x = -1, x = 1$.

Region where the curve lies:

	$x < -1$	$-1 < x < 1$	$x > 1$
$(x-1)^2$	$+$	$+$	$+$
$(x+1)$	$-$	$+$	$+$
y	$-$	$+$	$+$

B1

Sketch of the curve



B1 B1

14 (a).

$$6\sin x - 3\cos x \equiv R\sin(x - \alpha) = R\sin x\cos\alpha + R\cos x\sin\alpha$$

B1

By comparison,

$$R\cos\alpha = 6 \quad \to (1a), \qquad R\sin\alpha = 3 \quad \to (1b)$$

	$(1b) \div (1a)$ gives: $$\frac{R\sin\alpha}{R\cos\alpha} = \frac{3}{6}\ , \qquad \Rightarrow \tan\alpha = 0.5, \qquad \Rightarrow \alpha = 26.57°$$ $$R = \sqrt{3^2 + 6^2} = \sqrt{45}$$ $$\Rightarrow 6\sin x - 3\cos x \equiv \sqrt{45}\sin(x - 26.57°)$$ Maximum value: $$\{6\sin x - 3\cos x\}_{max} = \sqrt{45} \times 1 = \sqrt{45} \approx 6.708$$ (b). $$L.H.S = \frac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°} = \frac{1 + \tan 11°}{1 - \tan 11°} = \frac{\tan 45° + \tan 11°}{\tan 45° - \tan 11°}$$ $$= \tan(45 + 11)° = \tan 56°$$ (c). $$L.H.S = \sin B + \sin C - \sin A$$ $$= \left[2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)\right] - 2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)$$ For angles of a triangle, A, B, C, $$\sin\left(\frac{B+C}{2}\right) = \sin\left(90 - \frac{A}{2}\right) = \cos\left(\frac{A}{2}\right)$$ $$\cos\left(\frac{B+C}{2}\right) = \cos\left(90 - \frac{A}{2}\right) = \sin\left(\frac{A}{2}\right)$$ $$\Rightarrow L.H.S = \left[2\cos\left(\frac{A}{2}\right)\cos\left(\frac{B-C}{2}\right)\right] - 2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)$$ $$= 2\cos\left(\frac{A}{2}\right)\left[\cos\left(\frac{B-C}{2}\right) - \sin\left(\frac{A}{2}\right)\right]$$ $$= 2\cos\left(\frac{A}{2}\right)\left[\cos\left(\frac{B-C}{2}\right) - \cos\left(\frac{B+C}{2}\right)\right]$$ $$= 2\cos\left(\frac{A}{2}\right)\left[-2\sin\left(\frac{B}{2}\right)\sin\left(-\frac{C}{2}\right)\right]$$ $$= 4\cos\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right)$$	**B1** –for α **B1** –for R **B1** **B1** **B1** **B1** **B1** **B1** **B1**
15	(a). $$(2+5i)^2 + 5\frac{(7+2i)}{3-4i} - i(4-6i)$$ $$= 4 + 20i - 25 + \frac{(35+10i)(3+4i)}{9+16} - 4i - 6$$ $$= 16i - 27 + \frac{105 + 140i + 30i - 40}{25}$$ $$= \frac{(400i - 675) + (65 + 170i)}{25}$$ $$= \frac{570i - 610}{25} = \frac{114i}{5} - \frac{122}{5} = 22.8i - 24.4$$ (b). $$\frac{z-1}{z-i} = \frac{(x-1)+yi}{x+(y-1)i} = \frac{\{(x-1)+yi\}\times\{x-(y-1)i\}}{\{x+(y-1)i\}\times\{x-(y-1)i\}}$$	**B1** **B1** **B1** **B1** **B1** **B1**

	$$=\frac{(x-1)x-(x-1)(y-1)i+xyi+y(y-1)}{x^2+(y-1)^2}$$ $$=\frac{x^2-x-(xy-x-y+1)i+xyi+y^2-y}{x^2+(y-1)^2}$$ $$=\frac{(x^2+y^2-x-y)-(-x-y+1)i}{x^2+(y-1)^2}$$ $$\text{real part}=\frac{(x^2+y^2-x-y)}{x^2+(y-1)^2}$$ $$\text{imaginary part}=\frac{-(-x-y+1)}{x^2+(y-1)^2}$$ $$Arg\left(\frac{z-1}{z-i}\right)=\tan^{-1}\left(\frac{\text{imaginary part}}{\text{real part}}\right)=\frac{\pi}{3}$$ $$\tan^{-1}\left(\frac{x+y-1}{x^2+y^2-x-y}\right)=\frac{\pi}{3}$$ $$\frac{x+y-1}{x^2+y^2-x-y}=\tan\frac{\pi}{3}=\sqrt{3}$$ $$x+y-1=\sqrt{3}(x^2+y^2-x-y)$$ $$x^2\sqrt{3}+y^2\sqrt{3}-x\left(1+\sqrt{3}\right)-y\left(1+\sqrt{3}\right)+1=0$$ The locus is a circle. By comparison with the general equation: $x^2+y^2+2gx+2fy+c=0$ $$2g=-\left(\frac{1+\sqrt{3}}{\sqrt{3}}\right)=-\left(\frac{3+\sqrt{3}}{3}\right),\qquad \Rightarrow g=-\left(\frac{3+\sqrt{3}}{6}\right)$$ $$f=g=-\left(\frac{3+\sqrt{3}}{6}\right)\approx-0.7887,\qquad c=\frac{1}{\sqrt{3}}$$ $$\text{centre}=(-g,-f)=\left(\frac{3+\sqrt{3}}{6},\frac{3+\sqrt{3}}{6}\right)$$ $$\text{radius}=\sqrt{g^2+f^2-c}=\sqrt{\left(\frac{3+\sqrt{3}}{6}\right)^2+\left(\frac{3+\sqrt{3}}{6}\right)^2-\frac{1}{\sqrt{3}}}$$ $$=0.8165\text{ units}$$	**B1** **M1** **A1** **B1** **M1** **A1**
16	(a). $$x^2\frac{dy}{dx}=x^2+xy+y^2$$ $$\text{but,}\qquad y=ux,\qquad \Rightarrow\frac{dy}{dx}=\left(u+x\frac{du}{dx}\right)$$ Substituting for y and $\frac{dy}{dx}$ gives: $$x^2\left(u+x\frac{du}{dx}\right)=x^2+ux^2+u^2x^2$$ $$ux^2+x^3\frac{du}{dx}=(1+u+u^2)x^2$$	**B1** **M1**

	$$u + x\frac{du}{dx} = 1 + u + u^2$$ $$x\frac{du}{dx} = 1 + u^2$$ $$\int \frac{du}{1+u^2} = \int \frac{1}{x}dx$$ $$\tan^{-1} u = \ln x + c$$ $$\tan^{-1}\left(\frac{y}{x}\right) = \ln x + c$$	**M1** **A1**
	(b). Let h be the depth of the opening below the surface of the liquid at any time, t. Let h_0 be the initial depth of the opening below the surface of the liquid when the tank is full. $$\frac{dh}{dt} \propto \sqrt{h}$$ $$\frac{dh}{dt} = -kh^{\frac{1}{2}}$$ $$\int h^{-\frac{1}{2}}\,dh = -\int k\,dt$$ $$2\sqrt{h} = -kt + c$$ When $t = 0, h = h_0$ $$2\sqrt{h_0} = c$$ $$2\sqrt{h} = -kt + 2\sqrt{h_0}$$ When $t = 1, h = h_0 - 20$ $$2\sqrt{h_0 - 20} = -k + 2\sqrt{h_0}$$ $$-k = 2\sqrt{h_0 - 20} - 2\sqrt{h_0}$$ $$2\sqrt{h} = 2t\left(\sqrt{h_0 - 20} - \sqrt{h_0}\right) + 2\sqrt{h_0}$$ When $t = 2, h = h_0 - 20 - 19 = h_0 - 39$ $$2\sqrt{h_0 - 39} = 4\left(\sqrt{h_0 - 20} - \sqrt{h_0}\right) + 2\sqrt{h_0}$$ $$\sqrt{h_0 - 39} = 2\sqrt{h_0 - 20} - \sqrt{h_0}$$ $$h_0 - 39 = 4(h_0 - 20) - 4\sqrt{{h_0}^2 - 20h_0} + h_0$$ $$4\sqrt{{h_0}^2 - 20h_0} = 4h_0 - 41$$ $$16{h_0}^2 - 320h_0 = 16{h_0}^2 - 328h_0 + 1681$$ $$8h_0 = 1681$$ $$h_0 = 210.125 \text{ cm}$$	**B1** **B1** **B1** **B1** **M1** **M1** **M1** **A1**

END