S475/1 SUBSID. MATHEMATICS Paper 1 July 2017 $2\frac{2}{3}$ hours.



Uganda Advanced Certificate of Education MOCK SET 4 EXAMINATIONS 2017

SUBSIDIARY MATHEMATICS

Paper 1

2 hours 40 minutes

INSTRUCTIONS TO CANDIDATES

Answer all the **eight** questions in section **A** and only **four** questions in section **B**.

Any additional question(s) will not be marked.

Each question in section **A** carries **5** marks while each question in section **B** carries **15** marks.

All working must be shown clearly.

Graph paper is provided.

Where necessary, take acceleration due to gravity, $g = 9.8 \text{ m s}^{-2}$.

Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 MARKS)

(Answer all questions in this section.)

Qn 1: The arithmetic progression (A.P) is given below:

$$ln(2-x) + ln(3-x) + ln(5-x) + \cdots$$

Find the value of x. [5]

- **Qn 2:** Vector $\mathbf{a} = 3x\mathbf{i} + 4\mathbf{j}$ and $\mathbf{b} = x\mathbf{i} 3\mathbf{j}$ are perpendicular. Find the possible values of x. [5]
- **Qn 3:** Find the number of ways in which the letters of the word "ATTENDENCE" can be arranged if:
 - (i). the three E's must **not** be together.
 - (ii). Only two E's must be together. [5]
- **Qn 4:** Events *M* and *N* are such that $P(M' \cap N) = 0.3$, $P(M \cap N) = 0.55$, $P(M' \cap N') = 0.1$. Find:
 - (i). P(M),
 - (ii). P(M or N). [5]
- **Qn 5:** Solve the equation $\csc \theta \sec \theta 2 \cot \theta = 0$, for $0^{\circ} < \theta < 180^{\circ}$. [5]

Qn 6: A traveller finds out that the price index for breakfast (B), lunch (L) and supper (S) in Kampala and Mbarara were as shown in the table,

| Town | Price index | | | | | |
|---------|-------------|-----|-----|--|--|--|
| | В | L | S | | | |
| Kampala | 120 | 130 | 125 | | | |
| Mbarara | 115 | 135 | 110 | | | |

If the actual quantities consumed by the traveller for B, L and S were 300 g, 400 g and 300 g respectively. Calculate the weighted index for each town and comment on your result. [5]

- **Qn 7:** A continuous random variable X has a mean 15. The probability that X is less than 10 is 0.1057. Find the:
 - (i). variance to the nearest whole number.

(ii).
$$P(14 < X < 18)$$
. [5]

Qn 8: A boy pulls a box of mass 20 kg by means of a light inextensible string attached to it across a rough horizontal ground. The coefficient of friction between the box and the ground is 0.25. If the string is inclined at 30° to the horizontal and the box accelerates at 2 m s⁻², find the tension in the string.

SECTION B (60 MARKS)

(Answer any **four** questions from this section.)

Question 9:

The ages in years of teachers in a certain school were recorded as follows:

| 46 | 48 | 40 | 59 | 53 | 23 | 39 | 31 | 34 | 61 | 54 | 54 |
|----|----|----|----|----|----|----|----|----|----|----|----|
| 45 | 51 | 33 | 37 | 37 | 27 | 28 | 45 | 48 | 39 | 29 | 23 |
| 48 | 37 | 39 | 33 | 25 | 31 | 48 | 40 | 53 | 51 | 46 | 45 |
| 56 | 59 | 40 | 43 | 46 | 38 | 29 | 52 | 54 | 34 | 23 | 41 |
| 52 | 42 | 50 | 55 | 60 | 45 | 45 | 56 | 59 | 49 | 44 | 36 |
| 25 | 38 | 56 | 36 | 42 | 47 | 50 | 54 | 59 | 47 | 58 | 57 |

- (a). Construct a grouped frequency table with uniform class width of 5 starting with 20 as the lowest class limit. [3]
- (b). Calculate the:
 - (i). Mean age.
 - (ii). Modal age.
 - (iii). Standard deviation.

[7]

- (c). Draw a cumulative frequency curve and use it to estimate
 - (i). the median.
 - (ii). The number of teachers who should retire if the retirement age is 55 years. [5]

Question 10:

The table below shows the monthly sales of a certain product in (shs "000") for the year 2016.

| Month | Sales | Month | Sales |
|----------|-------|-----------|-------|
| January | 220 | July | 175 |
| February | 210 | August | 186 |
| March | 200 | September | 176 |
| April | 207 | October | 170 |
| May | 196 | November | 159 |
| June | 189 | December | 168 |

- (a). Calculate 6-point moving totals and hence the moving averages. [6]
- (b). (i). Plot on the same axes actual sales and moving averages. Comment on the trend of sales during the year.
 - (ii). Determine the sales in January 2017.

[9]

Question 11:

Two variables X and Y were recorded as shown below:

| X | 10 | 140 | 120 | 100 | 80 | 70 | 40 | 10 |
|---|-----|-----|-----|-----|----|----|----|-----|
| Y | 150 | 30 | 30 | 50 | 70 | 70 | 90 | 120 |

- (a). Plot a scatter diagram for the data and comment on the relationship between X and Y. Find X when Y = 75. [8]
- (b). Calculate a rank correlation coefficient and comment on the value obtained. [7]

Question 11:

- (a). Given the matrices $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} -1 & 1 \\ 3 & -5 \end{pmatrix}$. Find:
 - (i). λ such that $|A \lambda I| = 0$; where I is a 2 × 2 identity matrix.
 - (ii). AB and BA and comment on the results.

[8]

(b). Mr. X bought a shirt and a tie at shs. 20,500 and Mr. Y bought two shirts and three ties at shs. 48,000. Form a pair of simultaneous equations for the purchases and use matrix method to determine the cost of a shirt and a tie. [7]

Question 13:

A differential function for a certain curve is given by $\frac{dy}{dx} = x - 1$, given that the curve passes through (0,0).

- (a). Determine the equation of the curve. [5]
- (b). Sketch the curve. [3]
- (c). Find the area enclosed between the x-axis and the curve. [3]
- (d). Differentiate $(3x^2 1)^5$; hence or otherwise evaluate:

 $\int_0^2 x (3x^2 - 1)^4 \, dx. \tag{4}$

Question 14:

A car of mass one tonne is travelling down an incline of $\sin^{-1}\left(\frac{1}{20}\right)$ against a constant resistance of 2500 N. At an instant 25 m from the lower end of the incline, it is travelling at a velocity 4 m s⁻¹ with the engine working at 30 kW. Find the:

- (i). acceleration down the incline.
- (ii). Velocity at the end of the incline.
- (iii). Power output of the engine required to take it up the incline at a constant velocity of 10 m s^{-1} against the same resistance. [15]

END



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Uganda Advanced Certificate of Education MARKING GUIDE FOR MOCK SET 4 EXAMINATIONS 2017

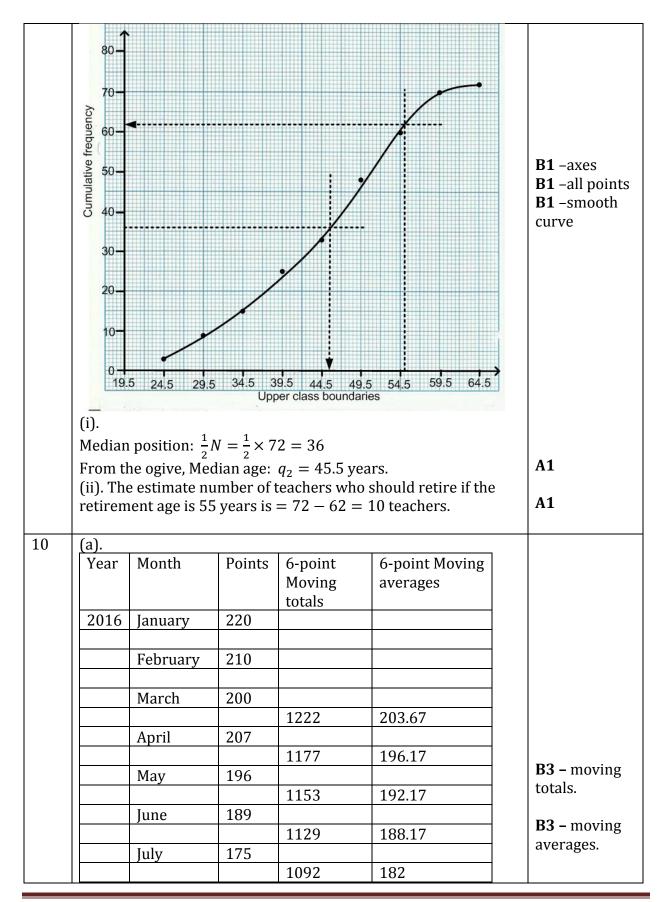
SUBSIDIARY MATHEMATICS Paper 1

Working SNo. Marks An A.P has a common difference, $\ln(3-x) - \ln(2-x) = \ln(5-x) - \ln(3-x)$ **M1** –equating the common difference $\ln\left(\frac{3-x}{2-x}\right) = \ln\left(\frac{5-x}{3-x}\right)$ M1 -using laws of logarithm $\left(\frac{3-x}{2-x}\right) = \left(\frac{5-x}{3-x}\right)$ **M1** (3-x)(3-x) = (2-x)(5-x)M1 -solving $3 - 3x - 3x + x^2 = 10 - 2x - 5x + x^2$ 3 - 6x = 10 - 7xx = 7**A1** 2 For perpendicular vectors, a.b = 0 $\begin{pmatrix} 3x \\ 4 \end{pmatrix} \cdot \begin{pmatrix} x \\ -3 \end{pmatrix} = 0$ M1 M1 dotting & equating to zero. $3x^2 - 12 = 0$ **B1** –output $x^2 = 4$ M1 -solving $x = \pm 2$ A1 -output 3 (i). Without restriction, "ATTENDENCE" has 10 letters with 2T's, 3E's and 2N's. number of arrangements = $\frac{10!}{T! E! N!} = \frac{10!}{2! 3! 2!} = 151200$ **B1** Taking 3E's as one such that they are always together, "ATTNDNC[EEE]" will have 8 letters with 2T's and 2N's number of arrangements $=\frac{8!}{T! N!} = \frac{8!}{2! 2!}$

| | If the three E's are not tegether | |
|---|---|-------------|
| | If the three E's are not together, number of arrangements = 151200 - 10080 = 141120 (ii). Taking 2E's as one such that they are always together, "ATTENDNC[EE]" will have 9 letters with 2T's, 2E's and 2N's | M1 A1 |
| | number of arrangements = $\frac{9!}{T! \text{ E! N!}} = \frac{9!}{2! \ 2! \ 2!} = 45360$ | M1 |
| | number of arrangements of $2E's = \frac{2!}{2!} = 1$ | |
| | total number of arrangements = $45360 \times 1 = 45360$ | A1 |
| 4 | (i). $P(M') = P(M' \cap N) + P(M' \cap N') = 0.3 + 0.1 = 0.4$ $P(M) = 1 - P(M') = 1 - 0.4 = 0.6$ (ii). | B1 M1 A1 |
| | $P(N) = P(M \cap N) + P(M' \cap N) = 0.55 + 0.3 = 0.85$ $P(M \text{ or } N) = P(M \cup N) = P(M) + P(N) - P(M \cap N)$ $= 0.6 + 0.85 - 0.55 = 0.9$ | M1 A1 |
| 5 | $ \frac{1}{\sin \theta} \times \frac{1}{\cos \theta} - 2 \cot \theta = 0 $ $ \frac{1}{\sin \theta} \times \frac{1}{\cos \theta} - \frac{2 \cos \theta}{\sin \theta} = 0 $ $ \frac{1}{\sin \theta \cos \theta} - \frac{2 \cos \theta}{\sin \theta} = 0 $ | B1 |
| | $\frac{1}{\sin \theta \cos \theta} = \frac{2 \cos \theta}{\sin \theta}$ $\sin \theta = 2 \sin \theta \cos^2 \theta$ $\sin \theta - 2 \sin \theta \cos^2 \theta = 0$ | B1 |
| | $\sin \theta (1 - 2\cos^2 \theta) = 0$ $\sin \theta = 0, \text{or,} (1 - 2\cos^2 \theta) = 0$ $\text{for,} \sin \theta = 0, \theta = 0^\circ, 180^\circ$ $\text{for,} (1 - 2\cos^2 \theta) = 0, \cos \theta = \pm 0.5,$ | M1 M1 |
| | $\Rightarrow \theta = 60^{\circ}, 120^{\circ}$ $\therefore \theta = 60^{\circ}, 120^{\circ}$ | A1 |
| 6 | Weighted index for Kampala = $\frac{\sum \left(\frac{P_1}{P_0} \times 100 \times W\right)}{\sum W}$ | |
| | $= \frac{120 \times 300 + 130 \times 400 + 125 \times 300}{300 + 400 + 300}$ | M1 |
| | $= \frac{36000 + 52000 + 37500}{1000} = \frac{125500}{1000} = 125.5$ $\sum \left(\frac{P_1}{P_0} \times 100 \times W\right)$ | A1 |
| | Weighted index for Mbarara = $\frac{\sum (P_0 \times 100 \times W)}{\sum W}$ $= \frac{115 \times 300 + 135 \times 400 + 110 \times 300}{300 + 400 + 300}$ | M1 |

| | $= \frac{34500 + 54000 + 33000}{1000} = \frac{121500}{1000} = 121.5$ Comment: There was a 21.5% increase in the prices of consumed quantities in Mbarara. | A1 B1 |
|---|--|----------------------|
| 7 | (i). $P(X < 10) = 0.1057$ $P\left(Z < \frac{10 - 15}{\sigma}\right) = 0.1057$ $\frac{10 - 15}{\sigma} = -\phi^{-1}(0.1057)$ but, $\phi^{-1}(0.1026) = 0.26$ Value in Add column = $1057 - 1026 = 31$ $\phi^{-1}(0.1057) = 0.268$ $\frac{10 - 15}{\sigma} = -0.268$ $-5 = -0.268\sigma$ $\sigma = 18.657$ variance = $\sigma^2 = 18.657^2 = 348$ (nearest whole number) (ii). $P(14 < X < 18) = P\left(\frac{14 - 15}{18.657} < Z < \frac{18 - 15}{18.657}\right)$ $= P(-0.054 < Z < 0.161)$ $= \phi(0.054) + \phi(0.161) = 0.0215 + 0.0640$ $= 0.0855$ | M1 A1 M1 A1 |
| 8 | Resolving vertically, $R + T \sin 30^{\circ} = 20g$ $R + 0.5T = 20 \times 9.8$ $R = 196 - 0.5T$ $\Rightarrow f = \mu R = 0.25(196 - 0.5T) = 49 - 0.125T$ Resolving horizontally, $T \cos 30^{\circ} - f = 20a$ $T \cos 30^{\circ} - (49 - 0.125T) = 20 \times 2$ $T \cos 30^{\circ} - (49 - 0.125T) = 40$ | B1 B1 M1 M1 |

| | | A1 | | | | | | | |
|---|---|------------------------------------|--------|----------------|----------------------------------|---------------------------------|-----------------------------|--------------------------------------|-----------------------------|
| 9 | (a). | Tallies | f | x | fx | fx^2 | C.F | Class | |
| | 20- | /// | 3 | 22 | , | , , , | 3 | boundaries 19.5-24.5 | |
| | 24 25- | //// / | 6 | 27 | 66 | 1452 | 9 | 24.5-29.5 | |
| | 29 30- | //// / | 6 | 32 | 162 | 4374 | 15 | 29.5-34.5 | B1 –for classes |
| | 34 35- | ++++ | 10 | 37 | 192 | 6144 | 25 | 34.5-39.5 | B1 –for |
| | 39 | //// | 8 | 42 | 370 | 13690 | 33 | 39.5-44.5 | frequencies |
| | 44 45- | /// /// | 15 | 47 | 336 | 14112 | 48 | 44.5-49.5 | B1 –for $\sum fx$ |
| | 49 | //// //// | 10 | | 705 | 33135 | | 1110 1110 | B1 –for $\sum f x^2$ |
| | 50- 54 | //// | 12 | 52 | | | 60 | 49.5-54.5 | |
| | 55- | // //// | 10 | 57 | 624 | 32448 | 70 | 54.5-59.5 | |
| | 59 60- | //// // | 2 | 62 | 570 | 32490 | 72 | 59.5-65.5 | |
| | 64 Total | | 72 | | 124 3149 | 7688 145533 | | | |
| | (b). (i). | Mean | 200 | - 2 | $\frac{1}{2} - \frac{\sum f}{2}$ | $\frac{x}{x} = \frac{3149}{72}$ | – 43 | 736 | M1 A1 |
| | | $=44.5,\Delta$ | | | _ , | | | c = 49.5 - | |
| | 44.5 = M | | $=L_m$ | + (- | Δ_1 | c = 44.5 | $+\left(\frac{1}{7}\right)$ | $\left(\frac{7}{12}\right) \times 5$ | M1 |
| | | A1 | | | | | | | |
| | (iii). $\sigma = \sqrt{\frac{\sum_{i=1}^{N}}{2}}$ | M1 A1 | | | | | | | |
| | (c). | 4) | ٧ | , 2 | - (| · - / | | | |
| | | | | | | | | | |
| | | | | | | | | | |



| | August | 106 | | | |
|---------------------------|----------------|-----------------|----------------------|---------------------------|---|
| | August | 186 | 1055 | 175.83 | |
| | September | 176 | 1033 | 173.03 | |
| | September | 170 | 1034 | 172.33 | |
| | October | 170 | 1001 | 1.2.00 | |
| | | | 859 + X | 859 + X | |
| | November | 159 | | 6 | |
| | December | 168 | | | |
| 2017 | January | X | | | |
| (b). (i) | 1 | | | | |
| (0). (1) | 1 | | | | D 4 |
| 220· (000°) 210· | | | • | • 6-point moving averages | B1 – axes B1 – plotting of original |
| Monthly sales (shs "000") | \ \ <i>\</i> \ | \ | • | —• Raw data | data B1 – line for original data |
| hly sa | | 11. | | | B1 – plotting |
| Would 190 | | /*. | · 8 I | | of moving averages. |
| 180 | | \ | XY | | B1 – trend line for |
| 170 | - | | | | moving averages. |
| 160 | | | \sim | | B1 – tracing the next |
| | | | | | moving |
| 150 | - Mar - Feb | - Jun May | - Oct - Sept | Months of 2016 | averages |
| | is a decreasin | g trend ir | the average | sales during the year | B1 –comment |
| 2016. (ii). Fr | om the graph, | | | | |
| | 0 | | $\frac{+X}{5} = 168$ | | M1 |
| | | 859 + | $X = 168 \times 6$ | | |
| | | $\times 6 - 85$ | 9 = 149 thou | sand shillinngs | A1 |
| | | n January | 2017 is 149 t | thousand shillings (i.e. | |
| 149,00 | ιυ/=J. | | | | |
| | | | | | |

11

| | X | Y | D | D | d | d^2 | |
|-----------|-------------------------------------|--------------------------------|--------------------------|------------------|---|---|---|
| | 10 | 150 | <i>R_X</i> 7.5 | R_Y | 6.5 | 42.25 | B1 –rank X |
| | 140 | 30 | 1 | 7.5 | -6.5 | 42.25 | B1 -rank Y |
| | 120 | 30 | 2 | 7.5 | -5.5 | 30.25 | B1 - |
| | 100 | 50 | 3 | 6 | -3 | 9 | differences |
| | 80 | 70 | 4 | 4.5 | -0.5 | 0.25 | B1 $-\sum d^2$ |
| | 70 | 70 | 5 | 4.5 | 0.5 | 0.25 | |
| | 40 | 90 | 6 | 3 | 3 | 9 | |
| | 10 | 120 | 7.5 | 2 | 5.5 | 30.25 | |
| | $\sum x = 570$ | | 7.5 | | 5.5 | $\sum d^2 = 163.5$ | |
| | Zx = 370 | <u>Z</u> y =010 | | | | $\underline{L}u = 105.5$ | |
| | ∇_{x} | 570 | | | $\nabla u = 6$ | 310 | |
| | $\overline{x} = \frac{\Delta x}{x}$ | $=\frac{370}{2}=71.$ | 25, | $\overline{y} =$ | $\frac{\Delta y}{x} = \frac{\Delta y}{x}$ | $\frac{610}{8} = 76.25,$ | |
| | n | $8 \rightarrow (\overline{x})$ | v) – (| 71 74 | n | 8 | B1 -for $(\overline{x}, \overline{y})$ |
| 1 | ., | $\rightarrow (x,$ | y | / 1, /(| | | |
| | Ϋ́Υ | | | | | | |
| 160 |)- | | | | | | |
| | • | | | | | | B1 –axes |
| 140 | 1 | | | | | | B2 –all points |
| | | | | | | | B1 –plotting |
| 120 | | | | | | | (71,76) |
| (1) | | | | | | | B1 –line of |
| | | | | | | | best fit |
| 100 | 0= | | | | | | M1 - |
| | | • \ | | | | | attempting to |
| 80 |) _ | | (71.7 | 6) | | | find the value |
| | | | × | γ , | | | of X. |
| 60 | | | | | | | OI A. |
| 60 |)= | | | | | | |
| | | | | X | | | |
| 40 |) - | | | | \ | | |
| | | | | | \ | • | |
| 20 | | | | | | | |
| 10 | | | | | | | |
| | | | | | | | |
| C | 0 20 | 40 60 | 80 | 100 | 120 | 140 X | |
| | | | 111111111 | 1111111111 | 111111111111111111111111111111111111111 | 144 144 144 144 144 144 144 144 144 144 | |
| | | _ | | | | ween X and Y. | |
| | | when $Y =$ | 75, <i>X</i> | = 71. | | | A1 |
| (c). | | _ | | | | | |
| | _ 1 | $-\frac{6\sum d^2}{n(n^2-1)}$ | _ 1 | 6 × | 163.5 | 0.0464 | |
| | $\rho = 1$ | $-\frac{1}{n(n^2-1)}$ | · = 1 - | 8(8 | $\frac{1}{(2-1)} =$ | -0.9404 | M1 A1 |
| Cor | | nificant at 19 | | | | | B1 |
| | | / | - (| 3-1 | | · -) | |
| 12 (i). | | | | | | | |
| 12 (1). | | | | | | | |
| | | | | | | | |

| | $\mathbf{A} - \lambda \mathbf{I} = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$ | M1 |
|----|---|-------------|
| | $= {3 - \lambda \choose 2} $ $ \mathbf{A} - \lambda \mathbf{I} = 0$ $(3 - \lambda)(4 - \lambda) - 2 \times 1 = 0$ $12 - 3\lambda - 4\lambda + \lambda^2 - 2 = 0$ $10 - 7\lambda + \lambda^2 = 0$ | M1 |
| | $\lambda^2 - 7\lambda + 10 = 0$ sum = -7, product = 10, factors = -2, -5 | M1 |
| | $(\lambda - 2)(\lambda - 5) = 0$ $\lambda = 2, \text{or,} \lambda = 5$ | A1 |
| | (ii). $ \mathbf{AB} = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 3 & -5 \end{pmatrix} = \begin{pmatrix} -3+3 & 3-5 \\ -2+12 & 2-20 \end{pmatrix} = \begin{pmatrix} 0 & -2 \\ 10 & -18 \end{pmatrix} \\ \mathbf{BA} = \begin{pmatrix} -1 & 1 \\ 3 & -5 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} -3+2 & -1+4 \\ 9-10 & 3-20 \end{pmatrix} = \begin{pmatrix} -1 & 3 \\ -1 & -17 \end{pmatrix} $ | M1 A1 |
| | $BA = \begin{pmatrix} 3 & -5 \end{pmatrix} \begin{pmatrix} 2 & 4 \end{pmatrix} = \begin{pmatrix} 9 - 10 & 3 - 20 \end{pmatrix} = \begin{pmatrix} -1 & -17 \end{pmatrix}$ Comment: $AB \neq BA$. | M1 A1 B1 |
| | (b). let $x = \cos t$ of a shirt and $y = \cos t$ of a tie. x + y = 20500 2x + 3y = 48000 $\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 20500 \\ 48000 \end{pmatrix}$ | B1 B1 |
| | $\binom{x}{y} = \frac{1}{3-2} \binom{3}{-2} \binom{-1}{1} \binom{20500}{48000}$ | M1 |
| | $= \frac{1}{1} {61500 - 48000 \choose -41000 + 48000} = {13500 \choose 7000}$ $\therefore x = 13500, y = 7000$ | M1 |
| | A shirt costs shs. 13,500. A tie costs shs. 7,000. | A1 A1 |
| 13 | (a). $\frac{dy}{dx} = x - 1$ | |
| | $y = \int (x - 1) dx$ $y = \frac{1}{2}x^2 - x + c$ | M1 |
| | $y = \frac{1}{2}x^2 - x + c$ At the point (0, 0), $x = 0$ and $y = 0$. | B1 |
| | $\Rightarrow 0 = 0 - 0 + c, \qquad \Rightarrow c = 0$ $y = \frac{1}{2}x^2 - x$ | M1 B1 |
| | The equation of the curve is $y = \frac{1}{2}x^2 - x$. | A1 |
| | (b). Turning point For turning points, | |
| | $\frac{dy}{dx} = 0$ | |

$$x-1=0$$

$$x=1$$
when, $x=1$, $y=\frac{1}{2}\times 1^2-1=-0.5$
turning point is $(1,-0.5)$

Nature of turning point

$$\frac{dy}{dx} = x - 1$$
$$\frac{d^2y}{dx^2} = 1$$

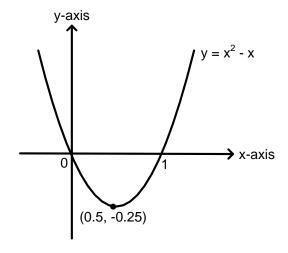
 \Rightarrow (1, -0.5) is a minimum point

Intercepts

when,
$$x = 0$$
, $y = 0 - 0 = 0$
when, $y = 0$, $\frac{1}{2}x^2 - x = 0$
 $x^2 - 2x = 0$, $\Rightarrow x(x - 1) = 0$
 $x = 0$, or, $x = 1$

Intercepts are: (0,0), (1,0).

Sketch



(c).
Area =
$$\int_0^6 (x^2 - 1) dx = \left[\frac{1}{3}x^3 - x\right]_0^1 = \left(\frac{1}{3} \times 1^3 - 1\right) - 0 = -\frac{2}{3}$$

Magnitude of Area = $\left|-\frac{2}{3}\right| = \frac{2}{3}$ sq. units

(d). $y = (3x^2 - 1)^5$

$$\Rightarrow \frac{dy}{dx} = 5 \times 6x \times (3x^2 - 1)^4 = 30x(3x^2 - 1)^4$$
$$\int_0^2 x(3x^2 - 1)^4 dx = \left[\frac{(3x^2 - 1)^5}{30}\right]_0^2 = \frac{(3 \times 2^2 - 1)^5}{30} - \frac{(0 - 1)^5}{30}$$

 $=\frac{161051}{30}-\frac{-1}{30}=\frac{26842}{5}=5368.4$

B1

B1

B1

B1

M1 M1 integration & substitution

A1

M1

B1

A1

| 14 | (i). $m = 1 \text{ tonne} = 1000 \text{ kg}, \theta = \sin^{-1}\left(\frac{1}{20}\right) = 2.87^{\circ}, P = 30 \text{ kW} = 30000 \text{ W}, u = 4 \text{ m s}^{-1}$ | |
|----|---|----------------------------|
| | R 1000gcos2.87° 1000g N 2.87° | B2 |
| | $P = F \times u, \qquad \Rightarrow F = \frac{P}{u} = \frac{30000}{4} = 7500 \text{ N}$ Resolving parallel to the plane gives, $F + 10000g \sin 2.87^{\circ} - 2500 = 1000a$ $7500 + 1000 \times 9.8 \times \sin 2.87^{\circ} - 2500 = 1000a$ $5490.686 = 1000a, \qquad \Rightarrow a = 5.491 \text{ m s}^{-2}$ (ii). $u = 4 \text{ m s}^{-1}, s = 25 \text{ m}, a = 5.491 \text{ m s}^{-2}$ $v^{2} = u^{2} + 2as = 4^{2} + 2 \times 5.491 \times 25 = 290.55$ $v = \sqrt{290.55} = 17.046 \text{ m s}^{-1}$ (iii). $v_{1} = 10 \text{ m s}^{-1}$ | B1 M1 M1 A1 M1 M1 M1 M1 A1 |
| | 1000gcos2.87° | B2 |
| | Resolving parallel to the plane gives, | |

| $F = 10000g \sin 2.87^{\circ} + 2500 = 1000 \times 9.8 \times \sin 2.87^{\circ} + 2500$ | M1 |
|---|-------|
| = 2990.686 N | B1 |
| $P_1 = F \times v_1 = 2990.686 \times 10 = 29906.86 \mathrm{W}$ | M1 A1 |
| | |

END