Name.. Combination..................

Index No..

P425/1
Pure Mathematics
Paper 1
July/August, 2019
3 Hours

ACEITEKA JOINT MOCK EXAMINATIONS 2019
UGANDA ADVANCED CERTIFICATE OF EDUCATION

PURE MATHEMATICS
PAPER 1
TIME: 3 HOURS

INSTRUCTIONS TO CANDIDATES:

- Answer **all** the **eight** questions in section **A** and only **five** questions in section **B**.
- Indicate the five questions attempted in section B in the table aside.
- Additional question(s) answered will **not** be marked.
- **All** working **must** be shown clearly.
- Graph paper is provided.
- Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

Question		Mark
Section A		
Section B		
Total		

SECTION A (40 marks)

Answer **ALL** *questions from this section. All questions carry equal marks.*

1. Solve the equation $5\sin 2x + 4 = 10\sin^2 x$ for $-\pi \le x \le \pi$. (5 marks)

2. The second and third terms of a geometric progression are 24 and $12(\alpha + 1)$ respectively. Find α if the sum of the first three terms of the progression is 76.
(5 marks)

3. The perpendicular bisector of the line joining the points $(3, 2)$ and $(5, 6)$ meets the x-axis at A and the y- axis at B, prove that the distance $B = 6\sqrt{5}$.
(5 marks)

4. Given that $= \frac{\sin x}{x}$, show that $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + xy = 0$. (5 marks)

5. Show that the lines L_1, vector equation $\underset{\sim}{r} = \begin{pmatrix}2\\5\end{pmatrix} + \lambda\begin{pmatrix}2\\-3\end{pmatrix}$ and L_2, vector equation $\underset{\sim}{r} = \begin{pmatrix}3\\-3\end{pmatrix} + \mu\begin{pmatrix}3\\2\end{pmatrix}$ are perpendicular and find the position vector of their point of intersection. (5 marks)

6. Find $\int \frac{\cos x}{4 + \sin^2 x}\, dx$. (5 marks)

7. Solve the equation $\log_x 32 - \log_{256} x = 1$. (5 marks)

8. A spherical water container of internal radius 10 cm has water to a maximum depth of 18 cm. Find the volume of the water in the container.
(5 marks)

SECTION B (60 marks)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

9. (a) Differentiate:

   (i) $\log_{10}\left(\frac{e^x}{\cos 3x}\right)$,

   (ii) $\sin^2(4x^2 + 5)$.

   (b) A curve is defined by the parametric equation $x = 2t^2$, $y = 4t - t^4$. Find the equation of the tangent to the curve at the point $(2, 3)$.
(12 marks)

10. (a) Given that $\frac{a}{b} = \frac{c}{d} = k$, show that $k = \frac{a+c}{b+d}$. Hence, solve the simultaneous equations

$$\frac{x+4z}{4} = \frac{y+z}{6} = \frac{3x+y}{5}$$
$$4x + 2y + 5z = 30.$$

(b) Solve the equation $e^{2x} - 4e^{x} + 3 = 0$.

(12 marks)

11. (a) Given that $\underset{\sim}{r}$ and $\underset{\sim}{s}$ are inclined at $60°$; $\underset{\sim}{t}$ is perpendicular to $\underset{\sim}{r} + \underset{\sim}{s}$ and $|\underset{\sim}{r}| = 8$, $|\underset{\sim}{s}| = 5$, $|\underset{\sim}{t}| = 10$, find $|\underset{\sim}{r} + \underset{\sim}{s} + \underset{\sim}{t}|$ and $|\underset{\sim}{r} - \underset{\sim}{s}|$.

(b) The equation of a plane $\boldsymbol{P}$ is $\underset{\sim}{r}.\begin{pmatrix} 2 \\ 6 \\ 9 \end{pmatrix} = 33$ where $\underset{\sim}{r}$ is the position vector of a point on $\boldsymbol{P}$. Find:

(i) The perpendicular distance from the origin to the plane.

(ii) The equation of a line $\boldsymbol{L}$ which passes through the point $A(5, -1, 2)$ and perpendicular to $\boldsymbol{P}$.

(iii) The coordinates of the points of intersection of $\boldsymbol{P}$ and $\boldsymbol{L}$.

(12 marks)

12. (a) Find $\int x \sec^2 x \, dx$.

(b) Evaluate $\int_2^3 \frac{3+3x}{x^3-1} \, dx$.

(12 marks)

13. (a) In the equation $ax^2 + bx + c = 0$, one of the root is the square of the other. Without solving the equation, prove that

$$c(a-b)^3 = a(c-b)^3$$

(b) (i). If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, obtain the equation whose roots are $\frac{1}{\alpha^3}$ and $\frac{1}{\beta^3}$

(ii). If in the equation in (b)(i) above, $\alpha\beta^2 = 1$, prove that $a^3 + c^3 + abc = 0$.

(12 marks)

14. (a) Find the equation of the line through the intersection of the lines $3x - 4y + 6 = 0$ and $5x + y + 13 = 0$ which

(i). passes through the point $(2, 4)$,

(ii). makes an angle of $60°$ with the x −axis.

(b) A circle touches the y −axis at a distance $+4$ from the origin and cuts off an intercept 6 from the x −axis. Find the equation of the circle.

(12 marks)

15. (a). Given that $\cot^2\theta + 3\,\text{cosec}^2\theta = 7$, show that $\tan\theta = \pm 1$.

(b). (i). Express the function $y = 4\cos x - 6\sin x$ in the form $R\cos(x + \alpha)$ where R is a constant and $0 \le \alpha \le 2\pi$. Hence find the coordinates of the minimum point of y.

(ii). State the values of x at which the curve cuts the y −axis.

(12 marks)

16. (a). Find the particular solution of the equation $\frac{dy}{dx} = x - \frac{2y}{x}$ given $y(2) = 4$.

(5 marks)

(b). The rate of increase of the population, P, of baboons in Busitema forest reserve is proportional to the number present in the forest at any time, t years. On 1st June 2010, there were 300 baboons in the forest and a year later, they were found to be 380.

(i). Form a differential equation involving P and t where t is time.

(1 mark)

(ii). Predict the population of baboons by first June 2018. (6 marks)

END