

ST. HENRY'S COLLEGE KITOVU

A'LEVEL APPLIED MATHEMATICS P425/2 SEMINAR QUESTIONS 2018

STATISTICS AND PROBABILITY

1. a) Events A and B are such that $P(A) = \frac{1}{3}$, $P(A \cap B) = \frac{1}{5}$ and $P(A^1 \cap B^1) = \frac{1}{6}$. Find;
- $P(A \cup B)$,
 - $P(A/B^1)$.
- b) There are 3 black and 2 white balls in each of 2 bags. A ball is taken from the first bag and put in the second bag. A ball is then taken from the second and put in the first bag. What is the probability that there is now the same number of black and white balls in each bag as there were to begin with?
2. a) The chance that Moses wins a game is $\frac{1}{3}$. If he plays nine games in a row, what is the;
- expected number of games,
 - chance of winning at least two games.
- b) At a bottle manufacturing factory, the new machine approximately makes 19% of the bottles that are damaged. If a random sample of 400 bottles is taken, find the probability that;
- more than 31 bottles will be damaged,
 - between 30 and 40 bottles inclusive will be damaged.
3. The table below show the frequency distribution of marks obtained by a group of students in a paper two mathematics examination.
- | Marks(%) | 10 – | 20 – | 35 – | 45 – | 65 – | 80 – | 90 – |
|-------------------|------|------|------|------|------|------|------|
| Frequency density | 1.8 | 2.4 | 5.8 | 3.3 | 1.2 | 0.4 | 0 |
- a) Calculate the;
- modal mark,
 - mean mark,
 - standard deviation,
 - number of students who scored above 54%.
- b) Draw a cumulative frequency curve and use it to estimate the;
- P_{10} - P_{60} range,
 - number of students who scored below 40%,
 - least mark if 20% of the students scored a distinction.

4. The table below shows the cost of ingredients used for making Chapattis for two different birthday parties for 2016 and 2017.

Ingredients	Cost	
	2016	2017
Salt	200	350
Baking flour	3800	4600
Cooking oil	1500	1800

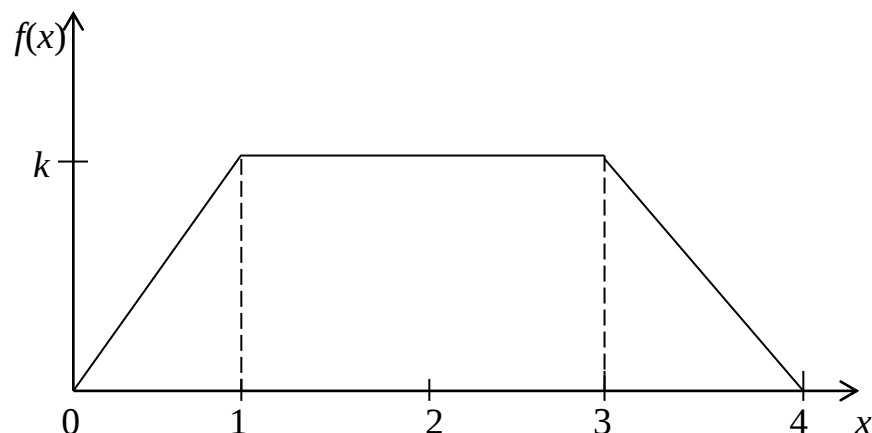
By taking 2016 as a base year, calculate the price relative for each ingredient and hence, obtain the average index number.

5. The table below shows marks scored by 8 students in mock and UNEB final examinations in Applied Mathematics.

Mock Examination	79	67	52	71	97	55	41	86
UNEB Final Examination	75	60	45	55	85	43	30	70

- Draw a scatter diagram for the data and comment on your result.
 - On the same diagram draw a line of best fit.
 - Use the line of best fit to estimate the mark that a student who scored 68 in Mock will score in UNEB.
 - Calculate the rank correlation coefficient for the marks in Mock and UNEB and comment on your result.
6. A random variable X takes the integer value x with $P(x)$ defined by
 $P(X = 1) = P(X = 2) = P(X = 3) = kx^2$, $P(X = 4) = P(X = 5) = P(X = 6) = k(7 - x)^2$.
- Find the;
- value of the constant k , hence sketch the graph of $f(x)$.
 - $E(Y)$ and $\text{Var}(Y)$ where $Y = 4X - 2$.
7. The continuous random variable X is such that $X \sim R(a, b)$. The lower quartile is 5 and the upper quartile is 9.
- Find;
- the values of a and b ,
 - $P(6 < X < 7)$,
 - the cumulative distribution function $F(x)$.

8. The probability distribution function of a continuous random variable X is represented as shown.



Find the;

- (i) mean of X ,
 - (ii) value of k ,
 - (iii) expression for the distribution,
 - (iv) $P(X \leq x)$, hence the median,
 - (v) $P(1.8 < x < 3.2)$.
9. The continuous random variable Y has a cumulative distribution function given by;

$$F(y) = \begin{cases} 0; & y < 1 \\ Ay^2(y^2 - 1); & 1 \leq y \leq 2 \\ 1; & y > 2 \end{cases}$$

Find the:

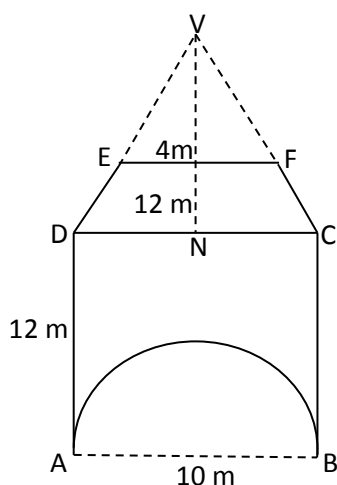
- (a) value of A ,
 - (b) 90th Percentile,
 - (c) $f(y)$, probability density function of Y ,
 - (d) $E(Y)$
 - (e) $\text{Var}(2Y+3)$
10. The positive error made by a machine while manufacturing metal strips is a random variable which can take up any value up to 0.5 cm. it is known that the probability of the length being not more than y centimeters ($0 \leq y \leq 0.5$) is equal to ky . Determine the;
- (a) value of k ,
 - (b) median positive error,
 - (c) probability distribution function,
 - (d) expected value of y ,
 - (e) standard deviation correct to 3.s.f.

11. On day one of the Coachella music festival, the height of the revelers can be modeled into a normal distribution of mean 1.75m and variance 0.0064m^2 . A draw is to be carried out and it is decided that one should have a height greater than 1.67m but less than 1.83m to participate.
- (a) Find the:
- (i) percentage of the people who qualify to take part in the draw.
 - (ii) fraction that is rejected because they are too tall.
- By day three of the event, the heights of the people present are normally distributed with mean, μ , and standard deviation, 0.085m . When the criteria used to select participants is not altered, 3.5% of the revelers are rejected because they are too short.
- (b) Find the:
- (i) value of μ ,
 - (ii) probability that a reveler whose height exceeds the mean qualifies to take part in the draw.
12. The germination time of a certain species of beans is known to be normally distributed. In a given batch of these beans, 20% take more than 6 days to germinate and 10% take less than 4 days.
- (a) Determine the mean and standard deviation of the germination time.
- (b) Find the 99.5% confidence limits of the germination time.

MECHANICS

13. (a) A particle has an initial position vector of $(4i + 3j - 7k)\text{m}$. The particle moves with a constant velocity of 6ms^{-1} parallel to $2i - j + 2k$. Find the position vector of the particle 3s later. Hence find how far the particle is from the origin.
- (b) A particle A of mass 2 kg moves under the action of a constant force $(2i + 4j + 3k)\text{N}$. At $t = 0\text{s}$, the particle is stationary and at a point with position vector $(4i - 10j + 7k)\text{m}$. Find the position vector of the particle at time $t = 5\text{s}$.
- 14.(a) An aircraft, at a height of 180m above horizontal ground and flying horizontally with a speed of 70ms^{-1} , releases emergency supplies. If these supplies are to land at a specific point, at what horizontal distance from this point must the aircraft release them?

- (b) A particle is projected from a point on a horizontal plane and has an initial speed of 42ms^{-1} . If the particle passes through a point above the plane, 70m vertically and 60m horizontally from the point of projection, find the possible angles of projection.
- 15.(a) A ball is projected from a point O with an angle of projection α . Find α if the horizontal range of the particle is five times the greatest height reached by it.
- (b) A stone thrown upwards from the top of a vertical cliff 56m high falls into the sea 4s later, 32m from the foot of the cliff. Find the speed and direction of projection. (Take g to be 10ms^{-2})
- 16.



ABCD is a uniform rectangular lamina with $AB = 10\text{m}$ and $AD = 12\text{m}$. A semi-circle of diameter AB is cut off. An isosceles uniform triangular lamina of base equal to CD, $EF = 4\text{m}$, $NV = 12\text{m}$ and EFV cut off is joined at side CD as shown above.

- (a) Calculate the centre of gravity of the remaining system.
- (b) If the system is then suspended at A, find the angle side AD makes with the downward vertical.
17. A non-uniform ladder AB of length 10m , weighing $5W$ and centre of mass 4m from A rest in a vertical plane with end B against a rough vertical wall and the end A against a rough horizontal surface. If the coefficient of friction at each end is $\frac{1}{4}$ and $\frac{1}{2}$ respectively.
- (a) A man of weight $13W$ begins to ascend the ladder from the foot, find how far he will climb before the ladder slips?
- (b) If a horizontal inextensible string is the attached from end A to the base of the wall, find the tension in the string when the man climbed the ladder to end B.

18. Two airfields A and B are 150km apart with B on a bearing of 045° from A. A wind of 30kmh^{-1} is blowing from a direction 260° . Assuming this wind remains constant throughout. Find the time required for aircraft to fly from A to B and back to A again, if the aircraft can fly at 100kmh^{-1} in still air.
19. A helicopter sets off from its base and flies at 50ms^{-1} to intercept a ship which, when the helicopter sets off, is at a distance of 5km on a bearing of 335° from the base. The ship is travelling at 10ms^{-1} on a bearing of 095° . Find;
- The course that the helicopter pilot should set if he is to intercept the ship as quickly as possible.
 - The time interval between the helicopter taking off and it reaching the ship.
20. At 11:45 am, a trawler is 10km due east of a launch. The trawler maintained a steady 10kmh^{-1} on a bearing 180° and the launch maintains a steady 20kmh^{-1} on a bearing 071° .
- Find the minimum distance the boats are apart in the subsequent motion, and the time at which this occurs.
 - Find to the nearest minutes, the length of time for which the two boats are within 8km of each other.
21. ABCD is a rectangle with $AB = 4\text{m}$ and $AD = 3\text{m}$. Forces of 9N , 8N , 4N , 7N and 10N act along AB, CB, CD, AD and BD respectively with the direction indicated by the order of the letters.
- Calculate the magnitude and direction of the single force that could replace this system of forces.
 - Find the equation of the line of action of the single force and where the line cuts AB.
- 22.(a) Find the coordinates of centre of gravity of a uniform lamina which lies in the first quadrant and is enclosed by the curves $y = 3x^2$, $y = 4 - x^2$ and the y – axis.
- Two rods AB and BC are joined together at B such that $\hat{ABC} = 50^\circ$. AB is uniform, of length 7m and mass 5kg . BC is uniform, of length 6m and mass 5kg . Find the distance of the centre of gravity from B.
- 23.(a) A small object of weight $4w$ in rough contact with a horizontal plane is acted upon by a force inclined at 30° to the plane. When the force is of magnitude $2w$, the object is about to slip. Calculate the magnitude of the normal reaction and coefficient of friction between the object and the plane.

- (b) A particle of mass $5kg$ is placed on a rough plane which is inclined at 30° to the horizontal. The angle of friction between the particle and the plane is 14° . Find the horizontal force that should be applied to the particle so that;
- The particle is just prevented from sliding down the plane.
 - The particle moves up the plane with an acceleration of $3.2ms^{-1}$.
24. A body A of mass $2kg$ is moving with a velocity $(-2i + 3j)ms^{-1}$ when it collides with a body B of mass $5kg$, moving with a velocity $(6i - 10j)ms^{-1}$. Immediately after the collision the velocity of A is $(3i - 2j)ms^{-1}$. Find;
- The velocity of B after the collision
 - The loss in kinetic energy of the system due to the collision
 - The impulse of A on B due to the collision.
- 25.(a) A particle is attached to one end of a light inextensible string which has its other end attached to a fixed point A. with the string taut, the particle describes a horizontal circle with a constant angular speed 2.8 rad s^{-1} , the centre of the circle being at a point O vertically below A. find the distance OA.
- (b) A vehicle is just on the point of slipping when parked on a bent that is banked at an angle of 20° to the horizontal.
- Find the coefficient of friction between the vehicle tyres and the surface of the road
 - If the vehicle were driven around this bend in a horizontal circular path of radius $60m$, find the greatest speed it could attain without slipping occurring.
26. A water pump raises $40kg$ of water a second through a height of $20m$ and ejects it with a speed of $45ms^{-1}$.
- Find the kinetic energy and potential energy per second given to the water
 - Calculate the effective rate at which the pump is working
27. With its engine working at a constant rate of $18kW$, a vehicle of mass 1.5 tonnes ascends a hill of 1 in 98 against a constant resistance to motion of $450N$. find;
- The acceleration of the vehicle up the hill when travelling with a speed of $10ms^{-1}$.
 - The maximum speed of the vehicle up the hill.
- 28.(a) A particle performs a SHM of periods $4s$ and amplitude $2cm$ about a centre O. find the time it takes the particle to travel from O to a point P, a distance $\sqrt{2} \text{ cm}$ from O.

- (b) A particle moves with SHM about a mean position O. when passing through two points which are $2m$ and $2.4m$ from O the particle has speeds of $3ms^{-1}$ and $1.4ms^{-1}$ respectively. Find the amplitude of the motion and the greatest speed attained by the particle.

NUMERICAL METHODS

- 29.(a) Given that $x = 2.40$, $y = 5.613$ and $z = 8.446$, each number rounded off to the given number of decimal places. Find the;

- (i) Limits within which the exact value of $\frac{x(4.5-z)}{y}$ lies
 (ii) Percentage error made in calculating $\frac{z-y}{x}$. (give your answer correct to 2dps)

- (b) Two decimal numbers X and Y were rounded off to give x and y with errors e_x and e_y respectively. Show that the maximum absolute and relative errors made in approximating $XY^{\frac{1}{2}}$ by $xy^{\frac{1}{2}}$ are given by $|y^{\frac{1}{2}}e_x| + \left|\frac{xe_y}{2y^{\frac{1}{2}}}\right|$ and $\left|\frac{e_x}{x}\right| + \frac{1}{2}\left|\frac{e_y}{y}\right|$ respectively.

- 30.(a) Use trapezium rule with six ordinates to estimate $\int_0^{\frac{\pi}{3}} \tan x \, dx$, correct to **three** decimal places.

- (b) Calculate the error made the estimate in (a) above and suggest how that error can be reduced.

- 31.(a) The table below shows the values of a continuous function f with respect to x.

x	1	2	3	4
$f(x)$	-1.632	-0.865	0.050	1.018

Using linear interpolation or extrapolation, find;

- (i) $f(x)$ when $x = 2.7$
 (ii) $f'(1.2)$.
 (b) Show that the root of the equation $f(x) = e^{-x} + x - 3$ lies between 2 and 3. Hence use linear interpolation to find the root correct to **two** decimal places

- 32.(a) Show that the root of the equation $\ln x + 2x - 3 = 0$ lies between 1 and 2.

- (b) Show that the iterative formula based on Newton Raphson's method for solving the equation is (a) above is given by

$$x_{n+1} = \frac{x_n(4 - \ln x_n)}{1 + 2x_n}, \quad n = 0, 1, 2, \dots$$

Hence find the root of the equation correct to **two** decimal places.

33.(a) Show that the Newton Raphson's formula for finding the fourth root of a number N is

$$x_{n+1} = \frac{1}{4} \left(3x_n + \frac{N}{x_n^3} \right), \quad n = 0, 1, 2, \dots$$

(b) Construct a flow chart that;

- Read N and the first approximation x_0
- Compute and print the root x_{n+1} and the number of iteration, n .

Using the flow chart, show that $(67)^{\frac{1}{4}} \approx 2.86$ to two decimal places.

END