KITEBI SECONDARY SCHOOL

BEGINNING OF TERM II EXAMINATIONS 2019

S6 PURE MATHEMATICS

Paper One

3 hours

INSTRUCTIONS

- Answer **all** questions in **Section A** and **five** questions from **Section B**.
- **All** working **must** be shown **clearly** and **neatly**.

SECTION A

Answer all the eight questions in this section.

1. Find all the values of x in the interval $180^0 \le x \le 540^0$ for which $\cot x + 5\cos ec^2 x = 6$. **(5 marks)**
2. Find the equation of the circle whose diameter is the line joining the points A(2,1) and B(6,5). **(5 marks)**
3. Using Binomial expansion, find the quadratic function that approximates to $f(x) = \frac{1}{\sqrt[3]{(1-3x)^2}}$ for values of x close to zero. Hence evaluate $\sqrt[3]{\frac{64}{25}}$ correct to 3 decimal places. **(5 marks)**
4. Given that $y = \text{In}\left(x + \sqrt{(x^2 + a)}\right)$ where a is a constant, prove that: $\frac{dy}{dx} = \frac{1}{\sqrt{x^2 + a^2}}$. Hence find $\frac{d^2y}{dx^2}$. **(5 marks)**
5. Evaluate: $\int_1^{\sqrt{5}} \frac{x^2}{\sqrt{x^4 - x^2}} dx$. **(5 marks)**
6. Find the vector equation of a line which passes through the point A(4, 3, -2) and parallel to the vector 2**i** + 3**j** + 4**k**. Where does this line meet the line $x = 0$? **(5 marks)**
7. Given that $\alpha^3\beta$ *and* $\alpha\beta^3$ are the roots of the equation $x^2 + 26x + 16 = 0$. From possible equations(s) whose roots are α *and* β. **(5 marks)**
8. Solve the differential equation: $\frac{y}{x}\frac{dy}{dx} = \frac{y^2 - 1}{x^2 - 1}$, given that $y = 3$ *when* $x = 2$. **(5 marks)**

SECTION B

9. (a) Prove that:

$$\frac{\sin 2x - 1 - \cos 2x}{2(1-\sin 2x)} = \frac{1}{\tan x - 1}$$ **(5 marks)**

(b) Solve for x in:

$$\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1} x.$$ **(7 marks)**

10. (a) The first term of a geometrical progression, G.P is $\sqrt{3}-1$ and the sum of the first three terms is $3\left(\sqrt{3}-1\right)$. Find the common ratio of the progression. **(5 marks)**

(b) Given that $U_1 = U_2$ and $U_{r+1} = \frac{1}{3}\left(2U_r^2 - 5\right)$. Find the possible values of U_1.

Hence find the value of $\sum_{r=1}^{5} U_{r+1}$ for each value of U_1 **(7 marks)**

11. (a) Given that $y = \tan^{-1}\left(\frac{\sin x + \cos x}{\sin x - \cos x}\right)$, show that $\frac{dy}{dx} + 1 = 0$. **(7 marks)**

(b) Determine $\frac{d}{dx}\left\{In\left(\frac{x}{\sqrt{1+x^2}}\right)\right\}$ when $x = 2.$ **(5 marks)**

12. (a) Evaluate $\int_2^4 \frac{1}{x(In\,x)^2}dx$. **(4 marks)**

(b) Find $\int 2e^x \sin x \cos x dx$. **(8 marks)**

13. (a) Vectors **a** = 2**i** – 2**j** – 2**k** and **b** = **i** – 3**j** + 2**k** form two sides of a triangle. Find its area. **(5 marks)**

(b) Given the lines **r** = 5**i** + 3**j** – 5**k** + λ (**i** + 2**j** – 3**k**) and $\frac{x-7}{3} = \frac{y+1}{-2} = \frac{z+4}{-2}$, find the coordinates of the point of intersection and the a cute angle between them. **(7 marks)**

14. (a) Use De Movires' theorem or otherwise to simplify:

$$\frac{(\cos\theta + i\sin\theta)(\cos 2\theta + i\sin\theta)}{\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)}.$$

(b) If z is a complex number, find the Cartesian equation and illustrate the locus given by $\left|\frac{z-1}{z+1-i}\right| < \frac{2}{3}$. **(12 marks)**

15. (a) Find the locus of point P which moves such that the sum of its distance from (2, 0) and (-2, 0) remains equal to $2\sqrt{5}$.

(b) Find the equation of the normal at $R(a\cos\theta, b\sin\theta)$ to an ellipse $b^2x^2 + a^2y^2 = a^2b^2$.
If the normal at R to the ellipse meets the x – axis at S. find the area of triangle ROS. **(12 marks)**

16. (a) Using small changes, evaluate $\sqrt[3]{28}$.

(b) Water starts running into an empty vessel at a rate of $6\pi\, cm^3/s$. The vessel is in the shape of the surface formed when the curve $4y = x^2$ is rotated completely about the $y-axis$. Show that when the depth of the water in the vessel is $2\pi y^2 cm^3$.
Find the rate at which the water level is rising when the water has been running for 3 seconds. **(12 marks)**

END