P425/1

PURE MATHEMATICS

PAPER 1

June/July 2017

3 hours

UACE RESOURCE MOCK EXAMINATIONS 2017

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in section* **A** *and* **five** *questions from section* **B**

Any additional question(s) answered will **not** *be marked*

All *working* **must** *be shown clearly*

Begin each question on a fresh page

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 MARKS)

1. (a) Simplify $\frac{(4.2^{n+1}-2^{n+2})}{(2^{n+1}-2^{n})}$ (03 marks)

   (b) Show that $\log(100-y^2)=2+\log(1-\frac{y^2}{100})$ (03 marks)

2. By using y = mx, solve the equations: $x^2-y^2 = 3$, $2x^2 + xy-2y^2 = 4$ (05 marks)

3. Prove that $4\tan^{-1}\left(\frac{1}{5}\right)-\tan^{-1}\left(\frac{1}{239}\right) = \frac{\pi}{4}$. (05 marks)

4. Differentiate $x\log_e x$ with respect to x hence evaluate $\int_1^2 \log_e x\, dx$ (06 marks)

5. The first three terms of in the expansion of $(1+kx)^n$ in ascending powers of x are

   $1-6x + \frac{33}{2}x^2$, find the values of k and n. (04 marks)

6. Find a vector perpendicular to the vectors **a = 2i - j + 3k** and **b = i + 2j + k**. (05 marks)

7. Water is emptied from a cylindrical tank of radius 20 cm at the rate of 2.5 litres per second and fresh water is added at the rate of 2 litres per second. Determine the rate at which the water level in the tank is changing.).
   (05 marks)

8. A is a point (0,4). P is a variable point such that it's distance from A is twice it's distance from the line 3x = 4y.Find the locus of P. (05 marks)

SECTION B (60 MARKS)

9. (a) Show that $i^9 + 2i^{11} + i^{13} = 0$. (04 marks)

   (b) If $z_1 = 1 + i\sqrt{3}$ and $z_2 = \sqrt{3} + i$, represent $\frac{z_1}{z_2}$ on an argand diagram. (03 marks)

(c) Given that $Z = 1 + i$ is a root of the equation $z^4 - 4z^3 + 3z^2 + 2z - 6 = 0$. Find the other roots. (05 marks)

10. (a) Angles A and B are both obtuse angles. Given that $\sin A = \frac{5}{13}$ and $\cos B = -\frac{3}{5}$, find $\tan(A-B)$. (05 marks)

(b) If $\tan\theta = \lambda\tan(A - \theta)$ show that $(\lambda-1)\sin A = (\lambda + 1)\sin(2\theta - A)$. (07marks)

11. The points P(1,3), R(4,-5) and Q(9,-1) are on the vertices of a triangle PQR.

Find the equation of the

(a) circle and hence state its radius and the coordinates of the centre. (08 marks)

(b) tangent to the circle at the point Q. (04 marks)

12. (a) Differentiate the following functions with respect to x.

(i) $\frac{\sqrt{x}}{x-2}$ (03 marks)

(ii) $2x^{x^2}$ (04 marks)

(b) Find the first three terms of the expansion $\frac{1}{x+1}$ using Maclaurin's theorem. (05 marks)

13. (a) Find the point of intersection between the lines $x - 2 = 2y + 1 = 3 - z$ and the plane $x + 2y + z = 3$. (04 marks)

(b) Show that the points with position vectors $\mathbf{OA} = 4\mathbf{i} - 8\mathbf{j} - 13\mathbf{k}$, $\mathbf{OB} = 3\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$ and $\mathbf{OC} = 3\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ are vertices of a triangle ABC. (04 marks)

(c) Find the equation of a plane through the origin parallel to the lines

$r_1 = \mathbf{3i + 3j - k} + m(i-j-2k)$ and $r_2 = \mathbf{4i - 5j - 8k} + t(3i+j-2k)$; m and t are scalars.

(04 marks)

14. (a) Express $\frac{2x^4-3x^3+7x^2-8x+5}{(x-1)^2(x^2+2)}$ into partial fractions. (07 marks)

(b) Hence find $\int\left(\frac{2x^4-3x^3+7x^2-8x+5}{(x-1)^2(x^2+2)}\right) dx$ (05 marks)

15. A curve is given by $y = \frac{(x+1)(x-3)}{x(x-2)}$.

(i) Show that for real x, y cannot be between 1 and 4. (04 marks)

(ii) Hence determine the turning points and distinguish them. (04 marks)

(iii) State the asymptotes and the intercepts of the curve. (03 marks)

(iv) Hence sketch the curve. (03 marks)

16. (a)Solve the differential equation: $x^2\frac{dy}{dx} = x^2 + xy + y^2$. (04 marks)

(b) When a murder is committed, the body originally at 37°C, loses heat at a rate proportional to the difference between the body temperature, H and the surrounding temperature, H_o. Suppose that after two hours the temperature is 35°C, and that the temperature of the surrounding air is a constant 20°C. If the body is found at 4:00 pm having a temperature of 30°C, estimate the when the murder was committed.

(08 marks)

END