S.6 HOLIDAY APPLIED MATHEMATICS WORK #2, 2020

Use g= 9.8m/s^2 where applicable unless otherwise.

SECTION A TYPE OF QUESTIONS.

1. Use trapezium rule with five strips to evaluate $\int_1^2 \log_{10} x^2 dx$, correct to three significant figures.

2. A particle initially located at point **A (2,4,6)** moves with a uniform speed of **12m/s** in the direction $2\boldsymbol{i} - 3\boldsymbol{j} - \boldsymbol{k}$. Find the distance of the particle from the origin.

3. The table below shows the cumulative frequency distribution (F) of the ages in years of the students in a given University.

Age	<20	<21	<22	<23	<24	<25	<26	<27
F	5	9	15	19	26	36	44	50

(a) Use the data to find the actual frequencies of the distribution.

(b) Draw an ogive and estimate the semi inter quartile range of the distribution.

4. A body of weight 50N is placed on a smooth plane inclined at an angle of $\sin^{-1}\left(\frac{1}{5}\right)$ to the horizontal. What would be the size of the minimum horizontal force required to maintain equilibrium? Find also the size of the normal reaction between the plane and the body.

5. A factory has a machine shop in which three machines A, B and C produce a sanitizer of capacity 100cm^3. A quality controller is equally likely to sample the sanitizer from A and B, and three times as likely to select sanitizer from C as he is from B. The defective rates from the machines are 10%,10% and 20% respectively. What is the probability that a sanitizer selected at random by the quality controller is defective?

6. The table below shows the behavior of x and y at different moments.

x	0.2	0.4	0.6
y	-1.609	-0.9163	-0.5108

Use linear interpolation or extrapolation to estimate the value of

(a) y when x= 0.72. (b) x when y= -1.9661.

7. A particle of mass 6kg rests on a rough plane inclined at angle of 50^0 to the horizontal. If the coefficient of friction between the particle and the plane is 0.32, calculate the force acting parallel to the plane which can move the particle up the plane.

8. Car P is accelerating at $2ms^{-2}$. At the point where its velocity is $10ms^{-1}$, it is overtaken by car Q moving at $57.6kmhr^{-1}$ and accelerating at $1ms^{-2}$. How long before does car P catches up with car Q?

9. A discrete random variable x has the following probability distribution.

x	1	2	3	4
P(X=x)	0.1875	0.5000	0.2500	0.0625

Calculate (a) $P(x \leq 3 / x \geq 2)$ (b) the median of x

10. The table below shows the prices (in Ug sh) and quantities of the items A, B and C for the years 2010 and 2011 respectively.

Item	**Prices for 2010**	**Prices for 2011**	**Quantities for 2010**	**Quantities for 2011**
A	20,000	18,500	6	4
B	25,200	22,400	8	9
C	30,600	19,500	7	6

Taking 2010 as the base period, calculate the weighted aggregate price index for 2011. Comment on your result.

11. Find the range within which the true value of $6.23(3.125 - \frac{10.135}{0.65})$.

12. An insect moves in the x-y plane so that its position vector at any time t is given by

$r = (4t^2 - 3t)\,\mathbf{i} + (8t+4-6t^2)\,\mathbf{j}$.

a. Find the distance covered after a time of 3 units.
b. Show that the insect is accelerating uniformly.

13. A box contains 4red and 3 blue balls. Three balls are chosen randomly from the box without replacement. Find the probability that;

a. the balls are all of the same colour.
b. the balls are blue given that the balls selected are of the same colour.

14. Use the graphical method to show the root of the equation $\mathbf{e}^{x} - 2x + 1 = 0$ between x=1.0 and x= 1.6. Hence estimate the root, correct to 1 decimal place.

15. Given that events M and N are mutually exclusive such that P(M)= ½ ,
P(either M or N) = 9/10.
Find (a) P($M' \cap N'$) (b) P($M' \cup N'$) (c) P($M \cup N'$) (d) P (N' / M).

SECTION B TYPE OF QUESTIONS

16. Six forces **8N, 10N, 12N, 14N, 16N** and **18N** act along the sides **AB, BC, CD, DE, EF** and **FG** respectively of a regular hexagon, their directions being shown by the order of the letters. If **AD** is horizontal,

a. Express each force as vector. (b) Find the magnitude and direction of the resultant force.

17. Use the graphical method to estimate the only positive root of the equation $2 - 2x - x^{3} = 0$, correct to 1 decimal place. Derive a simple formula based on Newton – Raphson method and use your graph result to find the better root of the equation, correct to three decimal places.

18. A particle of mass 200grams is acted on by forces (4t **i**+2 **j**) N, (-6 **i**+2t **k**) N and (6t **j**-**k**) N, where t is the time in seconds. If initially the velocity of the particle is (3 **i** – 2 **k**) m/s, its displacement is (6 **j** -4 **k**) metres .

19. Find the (a) impulse of the particle after 2 seconds. (b) position vector of the particle after 2 seconds.

20. A continuous random variable X has the probability functions given by;

$$f(x)=\begin{cases} kx; & 0 \le x \le 10 \\ k(20-x); & 10 \le x \le 20 \\ 0; & elsewhere \end{cases}$$

Where k is a constant.

a) Sketch f(x) and find the value of k.

b) Calculate P $[(0 < x < 8) / (5 < x < 12)]$

c) Construct a cumulative probability function, F(x) and use it to find the median.

21. Two balls are projected from the top of the vertical cliff 50m above the level ground. The balls are projected at the same time and in the same vertical plane, one at 25m/s and angle of elevation of $\tan^{-1}(3/4)$ and the other ball at 25m/s and angle of depression $\tan^{-1}(3/4)$.
Given that g= 10m/s^{2}, find the;

a. time interval between the balls hitting the ground.
b. horizontal distance between their points of impact.

22. (a) Use the trapezium rule with six ordinates to evaluate, $\int_0^1 \theta Sin\theta d\theta$ correct to three decimal places.

(b) Find the actual value of $\int_0^1 \theta Sin2\theta d\theta$,correct to three decimal places.

(c) Calculate the relative error made in (a) above and state how you would reduce such an error.

23. (a)Use trapezium rule with five sub intervals to find the value of $\int_1^3 x^2 Sin2xdx$, correct to three significant figures.

(b) Find the exact value of $\int_1^3 x^2 Sin2xdx$, correct to three significant figures.

(c) Find the percentage error made in (a) above and state how you would minimize the error.

24. The table below shows the masses (in grams) of 200 animals of the same species.

Mass	70-79	80-84	85-89	90-94	95-99	100-109
Cumulative Frequency	7	37	103	160	187	200

(a)Calculate the mean and variance.
(b)Draw a histogram and superimpose a frequency polygon.

25. (a)Round off to three significant figures;
(i) 6.9449 (ii)10.459 (iii)12436 (v) 0.01004

(b) The numbers A and B are estimated to a and b with errors e_1 and e_2 respectively.

Show that the maximum possible error made in estimating A^2/B is $\left|\frac{a^2}{b}\right|(2\left|\frac{e_1}{a}\right|+\left|\frac{e_2}{b}\right|)$.

Hence if a =2.5 and b =0.6, find the absolute error in a^2/b .

26. (a)A trunk is pulled across a horizontal ground by a force of 55N inclined at 35^0 to the horizontal. The trunk has a mass of 40kg and moves with a constant velocity. Calculate the resistance to motion. (ii) normal reaction.

(b) A lift has a mass of 600kg and holds three girls of masses 75kg, 45kg and 80kg. If $g = 10ms^{-2}$, determine the tension in the cable when the lift is moving upwards and decelerates at $0.5\ ms^{-2}$.

(c) A particle of mass 100grams is projected up a smooth inclined plane with an initial speed of 19.6m/s. The plane is inclined at 20^0 to the horizontal. Find the;

(i) deceleration of the particle.
(ii) distance travelled up the plane before the particle comes to rest.

27. (a) In a certain school, the need for pocket money is given as the reason for 40% of all theft cases in students` dormitories. If 8 theft cases are reported to the house mistresses, find the

(i) mean and standard deviation of the cases.
(ii) probability of reporting between 3 and 7 cases inclusive.
(iii) probability of reporting between 3 and 7 cases.

(b)A girls` chance of hitting the target is 0.2. Find the number of times the girl must try in order to be 95% sure that she hits the target at least once.

E N D