SECTION A (40MARKS)

Answer all questions in this section

1. A spherical balloon is being inflated so that at time when the radius is 5cm, the radius is increasing at the rate of 0.15cm/sec. Find the rate of increase of the volume at this time. ***(05marks)***

2. If A, B and C are angles of a triangle, prove that
$\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2\cos A \cos B \cos C$ ***(05marks)***

3. The sum of n terms of an A.P. is $2n+3n^2$; Find the n^{th} term. Hence find the sum of the 10^{th} and 20^{th} terms. ***(05marks)***

4. A curve passes through the origin and has gradient at any point (x, y) given by e^{2x-y}. Obtain the equation of the curve in the form $y = f(x)$ ***(05marks)***

5. Find the perpendicular (i.e, shortest) distance between the two skew lines:
$\vec{r} = (\hat{i} - 3\hat{j} + 3\hat{k}) + \lambda(-2\hat{i} + \hat{j} - 2\hat{k})$,
$\vec{r} = (3\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} - 3\hat{j} - 2\hat{k})$ ***(05marks)***

6. Evaluate the integral $\int \frac{x\sin^{-1}x}{\sqrt{(1-x^2)}}\,dx$ ***(05marks)***

7. Solve the simultaneous equations
$3^x - 3^{y+2} = 10$,
$2^y - 3^{x-2} = 2$ ***(05marks)***
(Give your answers correct to 4 significant figures)

8. Find the equations of the lines passing through the point of intersection of the lines $4x - 3y - 1 = 0$ and $2x = 5y - 3$, and equally inclined to the axes.
(05marks)

SECTION B (60 MARKS)

Answer any **five** *questions from this section.*
All *questions carry equal marks.*

9. (a) Find the Cartesian equation of the plane containing the line $\frac{x+7}{-3}=\frac{y+2}{-1}=\frac{z-4}{2}$ and perpendicular to the plane $6x+y+3z-17=0$. ***(07marks)***

   (b) Use vector properties to prove that, in any triangle ABC, $a^2=b^2+c^2-2bc\,\text{Cos}A$. ***(05 marks)***

10. (a) Show that $z=-3$ is a root of the equation $z^3-2z^2-8z+21=0$. Hence, find the other roots of the equation. ***(05marks)***

   (b) Use Demoivre's theorem to prove that the complex number $(\sqrt{3}-i)^n+(\sqrt{3}+i)^n$ is always real and hence find the value of the expression when $n=6$. ***(07 marks)***

11. (a) Find the equation of the normal at $P(ap^2, 2ap)$ on the parabola $y^2=4ax$. Show that L (a, 2a) passes through the point B (5a,-2a). Prove that there is one other point M on the parabola at which the normal passes through B and determine the coordinates of M. ***(12 marks)***

12. (a) Obtain the first three non-zero terms of the function $\tan^{-1}(2x)$ using maclaurin's theorem and hence evaluate $\int_0^1 tan^{-1}(2x)dx$, correct to 3 decimal places ***(06marks)***

   (b) Given that $x=\theta-\text{Sin}\theta, y=1-\text{Cos}\theta$, show that $y^2\frac{d^2y}{dx^2}+1=0$. ***(06marks)***

13. (a) A triangle has sides 13cm, 7cm and $5\sqrt{3}$cm. Without using tables or Calculators, find the smallest angle of the triangle and the diameter of the circumcirle. ***(08marks)***

   (b) Using the substitution $t=\tan\frac{1}{2}x$, solve the equation $2\text{Cos}x-\text{Sin}x+1=0$, for $-180^\circ \le x \le 180^\circ$ ***(04marks)***

14. (a) Show that $\log x + \frac{1}{2}\log\left\{1 + \frac{2y}{x} + \left(\frac{y}{x}\right)^2\right\} = \log(x + y)$ *(05 marks)*

(b) A customer opened up a bank account with Shs.500, 000. She deposits the same amount at the beginning of every year and makes no withdrawals. After how many years with her bank account balance first exceed ten million shillings if the bank account offers a 12.5% compound interest per annum? *(07 marks)*

15. If the curve $y = \frac{x^2-4x+4}{x+1}$,

(i) Show that the curve is restricted and state the region. Hence, or otherwise, investigate the nature of the turning points.

(ii) Determine the equation of asymptotes to the curve.

(iii) Sketch the curve *(12 marks)*

6. (a) Evaluate $\int_3^4 \frac{3x-2}{(x-2)(x^2+4)}\,dx$, correct to 4 decimal places *(06 marks)*

(b) A spherical bubble evaporates at a rate proportional to its surface area. If half of it evaporates in 3 hours, how long will the bubble disappear? *(06 marks)*