P425/2

S6 NUMERICAL METHODS

Test 3

1 *05marks*

The table below shows the cost **y** shillings for hiring a motor cycle for a distance **x** kilometers.

Distance (x km)	10	20	30	40
Cost (shs y)	2,800	3,600	4,400	5,200

Using linear interpolation or extrapolation to calculate

(i) the cost of lining a motor cycle for a distance of 45 km.

(ii) the distance mike travelled if he paid shs. 4,000

2 *05marks*

Given the numbers $X = 14.37$ and $Y = 2.586$, measured to their nearest number of decimal places indicated.

(i) determine the absolute error in $\frac{X}{Y}$.

(ii) the limit within which $\frac{X}{Y}$ lies, correct to 3 decimal places.

3 *05marks*

The numbers $a = 14.57$, $b = 2.991$ and $c = 82.1143$ are each rounded off to the given number of decimal places. Find the;

(i) limits with in which $c - ab$ lies correct to 3 decimal places.

(ii) percentage error in (i) above correct to 2 significant figures

4 *12marks*

a) Show that there exists a real root between 2.4 and 3 for $lnx = 4 - x$

b) Use linear interpolation twice to estimate the root to 3 significant figures

c) Hence using Newton –Raphson formula to estimate a better root for $lnx = 4 - x$ to 3 decimal places.

5 *12marks*

(a)(i) On the same axis, draw graphs of $y = x^2$ and $y = cosx$ for $0 \leq x \leq \frac{\pi}{2}$ at intervals of $\frac{\pi}{8}$.

(ii)From your graph, obtain to one decimal place, an approximate root of equation $x^2 - cosx = 0$.

(b) Using Newton Raphson method, find the root of the equation

	$x^2 - cosx = 0$. Taking the approximate root in (a) 0.5 an initial approximation. Give your answer to three decimal places.	
6	(a) Use the trapezium rule with six ordinates to estimate $\int_1^3 x^2 Inx\, dx$. Give your answer connect to 3 decimal places. (b) Hence find the percentage error made in your estimate and suggest how it can be reduced.	*12marks*
7.	(a) Given that ,$x = 2.40, y = 5.613$ and$z = 8.446$ each number rounded off to the given number of decimal places. Find the; (i) Limits within which the exact value of $\frac{x(4.5-z)}{y}$ lies (ii) Percentage error made in calculating $\frac{z-y}{x}$. (give your answer correct to 2dps) (b) Two decimal numbers X and Y were rounded off to give x and y with errors e_x and e_y respectively. Show that the maximum absolute and relative errors made in approximating $XY^{\frac{1}{2}}$by $xy^{\frac{1}{2}}$are given by $\left\|y^{\frac{1}{2}}e_x\right\| + \left\|\frac{xe_y}{2y^{\frac{1}{2}}}\right\|$ and $\left\|\frac{e_x}{x}\right\| + \frac{1}{2}\left\|\frac{e_y}{y}\right\|$ respectively.	
8.	(a)The table below shows the values of a continuous function f with respect to x.	*12marks*

x	1	2	3	4	
$f(x)$	-1.632	-0.865	0.050	1.018	

Using linear interpolation or extrapolation, find;
(i) $f(x)$ when $x = 2.7$
(ii) $f'(1.2)$
(b) Show that the root of the equation $f(x) = e^{-x} + x - 3$ lies between 2 and 3. Hence use linear interpolation to find the root correct to **two** decimal places