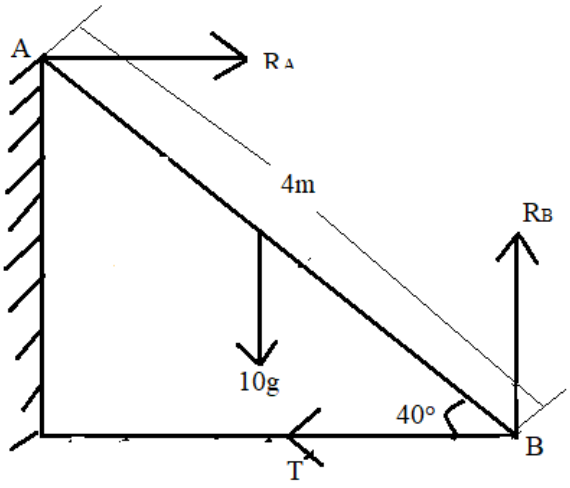
LADDER PROBLEMS

Ladder problems use the knowledge of moments and resolution of forces to work out problems.

Examples

1. A uniform ladder AB of mass 10kg and length 4m, rests with its upper end A against a smooth vertical wall and end B on a smooth horizontal ground. A light horizontal string which has one end attached to B and the other end attached to the wall keeps the ladder in equilibrium inclined at 40° to the horizontal. The vertical plane containing the ladder and the string is at right angles to the wall. Find the tension, T in the string and the normal reactions at the points A and B.



Resolving the forces in equilibrium:

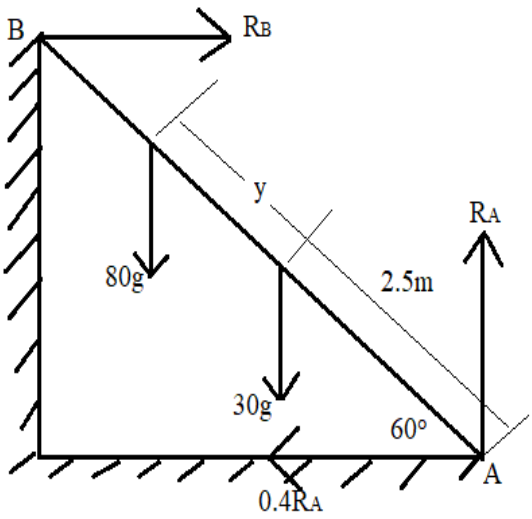
Vertically; Upward force equal to down force
R_B= 10g = 10x9.8 = 98N

Horizontally; Force in R.H.S = Force in L.H.S
T = R_A ……..(i)

Taking moments about point B;
R_A x 4Sin40° = 10g x 2Cos40°
R_A = 58.4N

From (i) T = 58.4N

2. A uniform ladder of mass 30kg and length 5m rests against a smooth vertical wall with lower end on rough ground, coefficient of friction 0.4. The ladder is inclined at 60° to the horizontal. Find how far a man of mass 80kg can ascend the ladder without slipping.



Let the man ascend a distance y from the foot of the ladder.
Resolving the forces in equilibrium:

Vertically;
R_A=80g + 30g = 110g ……..(i)

Horizontally;
R_B = 0.4R_A ……..(ii)

Taking moments about point A;
30g x 2.5Cos60 + 80g x yCos60° = R_B x 5Sin60° …….(iii)

Combining (i) and (ii) and substituting in (iii)
37.5g + 40gy = 0.4x110gx5Sin60°

$$y = \frac{220.xSin60 - 37.5}{40} = 3.83m$$

REVISION QUESTIONS

1. A uniform ladder of weight W and length $2l$ rests with one end on a smooth horizontal floor and the other end against a smooth vertical wall. The ladder is held in this position by a light horizontal inextensible string of length l which has one end attached to the bottom of the ladder and the other fastened to a point at the base of the wall vertically below the top of the ladder. Show that the tension in the string is $\frac{W}{2\sqrt{3}}$

2. A uniform ladder of weight, W leans against a rough vertical wall and stands on a rough horizontal surface. The coefficient of friction being equal at both ends of the ladder. If the ladder is in limiting equilibrium with $\mu = {}^{3}/_{5}$. Find the reactions at both ends.

3. A non-uniform ladder AB, 10m long and mass 8kg lies in limiting equilibrium with it lower end A resting on a rough horizontal ground and the upper end B resting against a smooth vertical wall. If the center of gravity of the ladder is 3m from the foot of the ladder and the ladder makes an angle of 30° with the horizontal, find the;

   (a) coefficient of friction between the ladder and the ground
   (b) reaction at the wall

4. A uniform ladder of weight W stands in limiting equilibrium on a rough horizontal surface and leans at 30° to a smooth vertical wall. Find the;
   (a) normal reactions at the ground and the wall
   (b) coefficient of friction at the ground hence find the total reaction at this point

5. A uniform ladder AB of mass 8kg and length 6m is resting in equilibrium at an angle of 50° to the horizontal with it's upper end A against a smooth vertical wall and its lower end B on a rough horizontal ground, coefficient of friction, μ. Find the normal reactions at A and B, frictional force and the least possible value of μ if the centre of gravity of the ladder is 2m from B.

6. A ladder AB rests with its end A on a rough horizontal floor and the other end B against a rough vertical wall, the coefficients of friction being μ_1 and μ_2 at the floor and on the wall respectively. If the centre of gravity of the ladder is at distances *a* and *b* from the ends A and B, show that when the ladder is in limiting equilibrium, the inclination of the ladder with the horizontal is $\tan^{-1}\left(\frac{a - b\,\mu_1\,\mu_2}{(a+b)\mu_1}\right)$.

7. A uniform ladder of length 2l and weight, W rests in a vertical plane with one end against a rough vertical wall and the other against a rough horizontal surface, the angles of friction at each end being $tan^{-1}\left(\frac{1}{3}\right)$ and $tan^{-1}\left(\frac{1}{2}\right)$ respectively.
   (a) If the ladder is in limiting equilibrium at either end, find the angle of inclination of the ladder to the horizontal
   (b) A man of weight 10 times that of the ladder begins to ascend it, how far will he climb before the ladder slips

8. A uniform rod is in equilibrium with its ends on a smooth plane inclined to the wall at 45°. Find the angle the rod makes with the horizontal.