P425/1
PURE MATHEMATICS
Paper 1
Nov. / Dec. 2020
3 hours

UGANDA NATIONAL EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in section* **A** *and any* **five** *questions from section* **B**.

Any additional question(s) answered will **not** *be marked.*

All *necessary working* **must** *be shown clearly.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A: (40 MARKS)

Answer **all** *the questions in this section.*

1. Solve the equation: $\sin x + \sin 2x + \sin 3x = 0$ for $0° \leq x \leq 180°$. *(05 marks)*

2. (a) Express $Z = \dfrac{3+i}{1-i}$ in the form $a + bi$, where a and b are integers. *(03 marks)*

   (b) Find the argument of Z. *(02 marks)*

3. Given that $y = \ln\left\{ x\sqrt{(x+1)^3} \right\}$, find $\dfrac{dy}{dx}$. *(05 marks)*

4. A plane is perpendicular to the vector $\boldsymbol{r} = (\boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k})$ and contains the point $P(-2, 0, 4)$. Determine the equation of the plane. *(05 marks)*

5. Evaluate $\displaystyle\int_0^{\pi/3} \tan^2 \tfrac{1}{2}x \, dx$. *(05 marks)*

6. In how many ways can the letters of the word BUNDESLIGA be arranged if;

   (a) there is no restriction? *(02 marks)*

   (b) the vowels must be together? *(03 marks)*

7. (a) Show that the curve whose parametric equations are $x = 9\cos\theta$ and $y = 12\sin\theta$ represents an ellipse. *(03 marks)*

   (b) Determine the eccentricity of the ellipse. *(02 marks)*

8. Find the gradient of the curve $x^2 \tan x - xy - 2y^2 = -2$ at the point $(0,1)$. *(05 marks)*

SECTION B: (60 MARKS)

Answer any **five** *questions from this section.* **All** *questions carry equal marks.*

9. (a) A polynomial $P(x)$ is given by $P(x) = (x + 2)(x - 1)Q(x) + (ax + b)$ where $Q(x)$ is the quotient and $ax + b$ is the remainder. When $P(x)$ is divided by $x - 1$, the remainder is 4 and when it is divided by $x + 2$, the remainder is 1. Find the values of a and b. *(05 marks)*

(b) (i) Expand $(1 + x^4)^{-1/2}$ up to the fourth term.

(ii) Use the first two terms of the expansion to find the value of

$$\frac{1}{\sqrt{144.0144}}$$

correct to **two** significant figures. *(07 marks)*

10. A circle passes through the points (1, 3), (2, 2) and (5, 7).
Find the equation of the;

(a) circle. *(07 marks)*

(b) tangent to the circle at the point (1, 3). *(05 marks)*

11. Express $\dfrac{2 - x + x^2}{(1 + x)(1 - x)^2}$ in partial fractions.

Hence evaluate $\displaystyle\int_0^{1/2} \frac{(2 - x + x^2)}{(1 + x)(1 - x)^2}\,dx$ correct to **three** decimal places.

(12 marks)

12. (a) Determine the angle between the vectors
$\boldsymbol{p} = \boldsymbol{i} + 9\boldsymbol{j} + 4\boldsymbol{k}$ and $\boldsymbol{q} = \boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$. *(05 marks)*

(b) The vector equations of two lines are $\boldsymbol{r}_1 = \boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k} + \lambda(-\boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k})$ and $\boldsymbol{r}_2 = -2\boldsymbol{j} + 5\boldsymbol{k} + \mu(\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k})$.
Find the coordinates of the point of intersection of the two lines.

(07 marks)

13. Express $8\sin\theta - 15\cos\theta$ in the form $R\sin(\theta - \alpha)$ where R and α are constants.
Hence find the maximum value of $8\sin\theta - 15\cos\theta$ and the smallest positive value of θ at which the maximum occurs. *(12 marks)*

14. (a) Solve the simultaneous equations:

$$2x^2 - 5xy + 2y^2 = 0$$

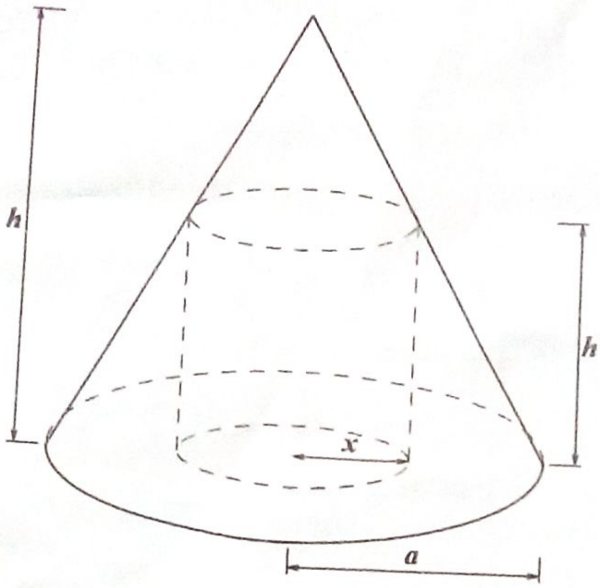
$$x + y = 6$$

(05 marks)

(b) If α and β are the roots of the equation $x^2 - px + q = 0$, find the equation whose roots are $\dfrac{\alpha^2 - 1}{\alpha}$ and $\dfrac{\beta^2 - 1}{\beta}$.

(07 marks)

15. The diagram below shows a cone of radius a and height h, in which a cylinder of radius x is inscribed.



(a) Express the height h' of the cylinder in terms of x, a and h.

(04 marks)

(b) Show that $V' = \frac{4}{9} V$ where V' is the greatest volume of the cylinder that can be inscribed in the given cone of volume V. *(08 marks)*

16. (a) Solve the differential equation

$$\frac{ds}{dt} = \frac{2e^{2t}}{\sqrt{S}} \text{ given that } S = 9 \text{ when } t = 0.$$

(b) Determine the value of;

(i) t when $S = 16$

(ii) S when $t = 2.4$

(Give your answer to **2** significant figures) *(12 marks)*