UACE MATHEMATICS PAPER 2 2016 guide

SECTION A (40 marks)

Answer all questions in this section

1. A ball s projected vertically upwards and returns to its point of projection 3seconds later. Fin the
   (a) speed with which the ball is projected
   (b) the greatest height reached.
2. The table shows the values of two variables P and Q

P	14	15	25	20	15	7
Q	30	25	20	18	15	22

   Calculate the rank correlation coefficient between the two variables.
3. Use the trapezium rule with 4 sub-divisions to estimate $\int_0^{\frac{\pi}{2}} \cos x \, dx$.
   Correct to **three** decimal places
4. A body of mass 4kg is moving with initial velocity $5ms^{-1}$ on a plane. The kinetic energy of the body is reduced by 16 joules in a distance of 40m. Find the deceleration of the body.
5. A continuous random variable X has a cumulative distribution function:

$$F(x) = \begin{cases} 0, x \leq 0 \\ \lambda x2, 0 \leq x \leq 4 \\ 1, x \geq 4 \end{cases}$$

   Find the
   (a) Value of λ
   (b) Probability density function (x)
6. The table below shows values of f(x) foe the given values of x.

X	0.4	0.6	0.8
f(x)	-0.9613	-0.5108	-0.2231

   Use linear interpolation to determine $f^{-1}(-0.4308)$ correct to 2 decimal places.
7. A particle of mass 2kg rests in limiting equilibrium on a rough plane inclined at 30^0 to the horizontal. Find the value of coefficient of friction.
8. A bag contains 5 Pepsi cola and 4 Mirinda bottle tops. Three bottle tops are picked at random from the bag one after the other without replacement. Find the probability that the bottle tops picked are of the same type.

SECTION B (60 MARKS)

Answer any five questions from this section. All questions carry equal marks

9. The data below shows the length in centimetres of different calendars produced by the printing press. Accumulative frequency distribution was formed.

Length (cm)	<10	<30	<35	<40	<50	<60
Cumulative frequency	4	20	32	42	48	50

(a) Construct frequency distribution table
(b) Draw a histogram and use it to estimate the modal length
(c) Find the mean length of the calendars

10. Five forces of magnitude 3N, 4N, 4N, 3N and 5N act along the lines AB, BC, CD, DA and AC respectively, of a square of side 1m. The direction of the forces is given by the order of the letters. Taking AB and AD as reference axes, find the
(a) magnitude and direction of resultant forces.
(b) point where the line of action of the resultant force cuts the side AB.

11. Given the equation $X^3 - 6x^2 + 9x + 2 = 0$
(a) Show that the equation has a root between -1 and 0.
(b) (i) Show that the Newton Raphson formula approximating the root of the equation is given by $X_{n+1} = \frac{2}{3}\left[\frac{x_n^3 - 3x_n^2 - 1}{x_n^2 - 4x_n + 3}\right]$

(ii) Use the formula in (b)(i) above, with initial approximation of $x_0 = -0.5$, to find the root of the given equation correct to two decimal places

12. A newspaper vender by 12 copies of a sport magazine every week. The probability distribution for the number of copies sold in a week is given in the table below

Number of copies sold	9	10	11	12
probability	0.2	0.35	0.3	0.15

(a) Estimate the
(i) Expected number of copies that she sells in a week
(ii) Variance of the number of copies sold in a week.
(b) The vendor buys the magazine at shs. 1,200 and sells it at shs. 1,500. Any copies not sold are destroyed. Construct a probability distribution table for the vendor's weekly profit from the sales. Hence calculate her mean profit.

13. A particle starts from rest at a point (2, 0, 0) and moves such that its acceleration at any time t>0 is given by a = [16cos4ti + 8sin2tj + (sint – 2sin2t)k]ms^{-2}. Fins the
(a) speed when t = $\frac{\pi}{4}$.
(b) distance from the origin when t = $\frac{\pi}{4}$

14. The numbers x and y are approximate by X and Y with errors Δx and Δy respectively
(a) Show that the maximum relative error in xy is given by $\left|\frac{\Delta x}{x}\right| + \left|\frac{\Delta y}{y}\right|$.
(b) If x = 4.95 and y = 2.013 are rounded off to the given number of decimal places, calculate
(i) percentage error in xy.
(ii) Limits within which xy is expected to lie, give your answer to three decimal places.

15. The drying time of any newly manufactured paint is normally distributed with mean 110.5 minutes and standard deviation 12 minutes.
    (a) Find the probability that paint dries between 104 and 109 minutes
    (b) If a random sample of 20 tins of paint was taken, find the probability that the mean drying time of the samples is between 108 and 112 minutes.
16. A particle of mass 2kg moving with Simple Harmonic Motion (SHM) along the x-axis, is attracted towards the origin 0 by a force of 32x Newton. Initially the particle is at rest x = 20. Find the
    (a) Amplitude and period of oscillation
    (b) Velocity of the particle at any time, t>0
    (c) Speed when t = $\frac{\pi}{4}$ seconds

Solutions

SECTION A (40 marks)

Answer all questions in this section

1. A ball s projected vertically upwards and returns to its point of projection 3seconds later. Fin the
    (a) speed with which the ball is projected

    time taken by the ball to reach maximum point = $\frac{3}{2} = 1.5s$

    v = u – gt at maximum height, v= 0

    0 = u – 9.8 x 1.5

    u = 9.8 x 1.5 = 14.7ms^{-1}
    (b) the greatest height reached.

    H = ut – ½t^2

    = 14.7 x 1.5 – ½ (1.5)^2 = 11.025m
2. The table shows the values of two variables P and Q

P	14	15	25	20	15	7
Q	30	25	20	18	15	22

Calculate the rank correlation coefficient between the two variables.

Using Spearman's approach

P	Q	R_P	R_Q	d	d^2
14	30	5	1	4	16
15	25	3.5	2	1.5	2.25
25	20	1	4	-3	9
20	18	2	5	-3	9
15	15	3.5	6	-2.5	6.25
7	22	6	3	3	9
					$\sum d^2 = 51.5$

p = 1 - $\frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6\,x\,51.5}{6(35-1)} = 1 - \frac{309}{210}$ = -0.4714

Using Kendall's approach

Using agreements and disagreements

R_P	R_Q	A	D	A-D
1	5	1	4	-3
2	3.5	1	2	-1
3	6	0	3	-3
4	1	2	0	2
5	2	1	0	1
6	3.5	-	-	S = -4

$\tau = \frac{2S}{n(n-1)} = \frac{2\,x{-}4}{6(6-1)} = \frac{-8}{30} = 0.2667$

Using crossing

Let the variables be A, B, C, D, E, F

P	RP		Q	RQ	
25	1	C	30	1	A
20	2	D	25	2	B
15	3.5	B	22	3	F
15	3.5	E	20	4	C
14	5	A	18	5	D
7	6	F	15	6	E

C = 9

$\tau = 1 - \frac{4C}{n(n-1)} = 1 - \frac{4\,x9}{6(6-1)} = 1 - \frac{36}{30} = -0.2$

3. Use the trapezium rule with 4 sub-divisions to estimate $\int_0^{\frac{\pi}{2}} \cos x\, dx$.

   Correct to **three** decimal places

   h = $\frac{\frac{\pi}{2}-0}{4} = \frac{\pi}{8}$

   let y = $\int_0^{\frac{\pi}{2}} \cos x\, dx$.

x	y	
0	1	
$\frac{\pi}{8}$		0.92388
$\frac{\pi}{4}$		0.70711
$\frac{3\pi}{8}$		0.38268
$\frac{\pi}{2}$	0	
Sum	2	2.01367

$\int_0^{\frac{\pi}{2}} \cos x\, dx. = \frac{h}{2}[(y_0 + y_4) + 2(y_1 + y_2 + y_3)]$

$= \frac{\pi}{16}[1 + 2(2.01367)]$

= 0.9871159 = 0.987 (3D)

4. A body of mass 4kg is moving with initial velocity 5ms^{-1} on a plane. The kinetic energy of the body is reduced by 16 joules in a distance of 40m. Find the deceleration of the body.

   Work done = change in K.E

   F x d =change in K.E

   ma x d = -16

   160a = -16

   a = $\frac{-16}{160} = -0.1ms^{-2}$

   ∴ the deceleration is 0.1ms^{-2}

Alternatively

Change in K.E = $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$

16 = $\frac{1}{2}(4)(v^2 - u^2)$

16 = 2

Change

5. A continuous random variable X has a cumulative distribution function:

F(x) = $\begin{cases} 0, x \leq 0 \\ \lambda x^3, 0 \leq x \leq 4 \\ 1, x \geq 4 \end{cases}$

Find the

(a) Value of λ

F(4) = 1

$\lambda(4^3) = 1$

64λ= 1

λ = $\frac{1}{64}$

(b) Probability density function (x)

For x ≤ 0, f(x) = 0

For x ≥ 4, f(x) = $\frac{d}{dx}(4) = 0$

For 0≤ x ≤ 4.

f(x) = $\frac{d}{dx}(\lambda x^3)$ = 3λ$x^2 = \frac{3}{16}x^2$

$\therefore f(x) = \begin{cases} \frac{3}{16}x^2 & 0 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$

6. The table below shows values of f(x) foe the given values of x.

X	0.4	0.6	0.8
f(x)	-0.9613	-0.5108	-0.2231

Use linear interpolation to determine f^{-1}(-0.4308) correct to 2 decimal places.

Extract

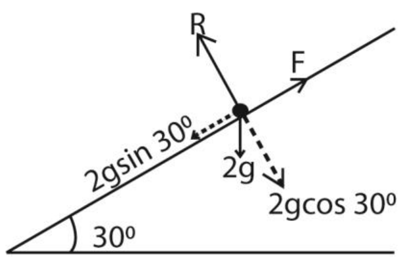
x	0.6	x_1	0.8
f(x)	-0.5108	-0.4308	-0.2231

$\frac{-0.4308-(-0.5108)}{x_1-0.6} = \frac{-0.2231-(-0.5108)}{0.8-0.6}$

$\frac{0.08}{x_1-0.6} = \frac{0.2877}{0.2}$

x_1 = 0.66

7. A particle of mass 2kg rests in limiting equilibrium on a rough plane inclined at 30^0 to the horizontal. Find the value of coefficient of friction.



R = 2gcos 30^0
F = 2gsin 3^0
μR = 2gsin30^0
$\mu[2gcos\ 30^0]$ = 2gsin30^0
$\mu = \frac{2gsin\ 30^0}{2gcos\ 30^0}$ = tan 30^0 = 0.57735

8. A bag contains 5 Pepsi cola and 4 Mirinda bottle tops. Three bottle tops are picked at random from the bag one after the other without replacement. Find the probability that the bottle tops picked are of the same type.
P(all the same type) = P($P_1 \cap P_2 \cap P_3$) + P($M_1 \cap M_2 \cap M_3$)
$= \frac{5}{9}\ x\ \frac{4}{8}\ x\ \frac{3}{7} + \frac{4}{9}\ x\ \frac{3}{8}\ x\ \frac{2}{7}$
$= \frac{60+24}{504}$
$= \frac{84}{504}$
= 0.1667

SECTION B (60 MARKS)

Answer any five questions from this section. All questions carry equal marks

9. The data below shows the length in centimetres of different calendars produced by the printing press. Accumulative frequency distribution was formed.

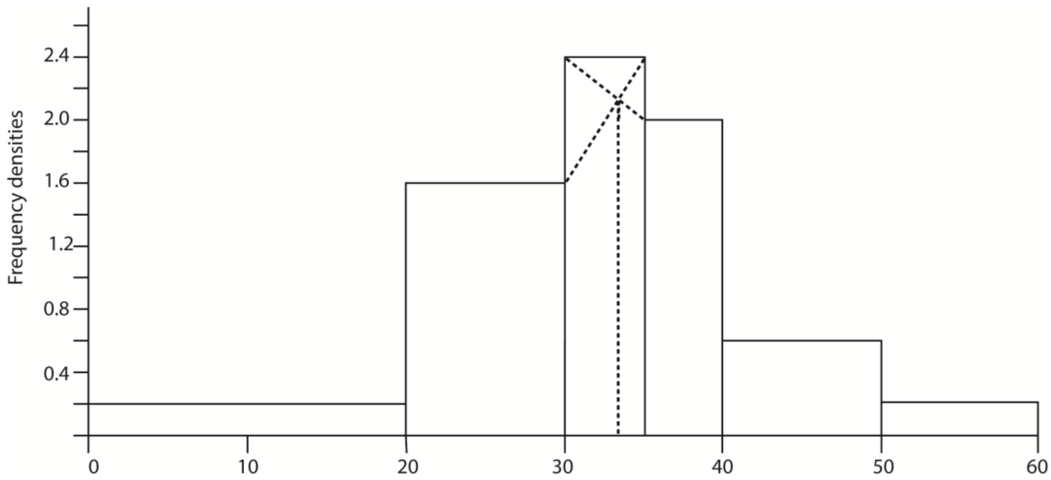
Length (cm)	<10	<30	<35	<40	<50	<60
Cumulative frequency	4	20	32	42	48	50

(a) Construct frequency distribution table

Length (cm)	F	cf
0 - < 20	4	4
20 - < 30	16	20
30 - < 35	12	32
35 - < 40	10	42
40 - < 50	6	40
50 - < 60	2	50
	$\sum f = 50$	

(b) Draw a histogram and use it to estimate the modal length

Length (cm)	x	c	f	cf	fx	fd
0 - < 20	10	20	4	4	40	0.2
20 - < 30	25	10	16	20	400	1.6
30 - < 35	32.5	5	12	32	390	2.4
35 - < 40	37.5	5	10	42	375	2.0
40 - < 50	45	10	6	40	270	0.6
50 - < 60	55	10	2	50	110	0.2
			$\sum f = 50$		$\sum fx = 1585$	



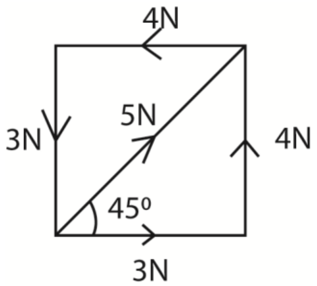
Mode = 33.5

(c) Find the mean length of the calendars

Mean = $\frac{\sum fx}{\sum f} = \frac{1585}{50} = 31.7$

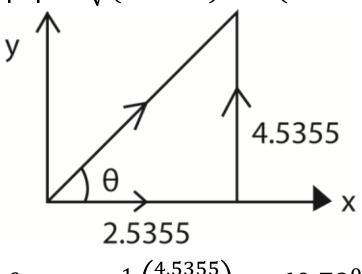
10. Five forces of magnitude 3N, 4N, 4N, 3N and 5N act along the lines AB, BC, CD, DA and AC respectively, of a square of side 1m. The direction of the forces is given by the order of the letters. Taking AB and AD as reference axes, find the

(a) magnitude and direction of resultant forces.



$\binom{x}{y} = \binom{3}{0} + \binom{0}{4} + \binom{-4}{0} + \binom{0}{-3} + \binom{5\cos 45^0}{5\sin 45^0} = \binom{2.5355}{4.5355}$

$|R| = \sqrt{(2.5355)^2 + (4.5355)^2}$ =5.1961N



$\theta = tan^{-1}\left(\frac{4.5355}{2.5355}\right) = 60.79^0$

(b) point where the line of action of the resultant force cuts the side AB.

Equation for line of action is given by

G – Xy + yx = 0

A: G = 4 x 1+ 4 x 1 = 0

G = 4 + 4 = 8Nm

By substitution

8- x(4.5355) + y(2.5355) = 0

8 = 4.5355x + 2.5355y = 0

The line cuts AB when y = 0

x = $\frac{8}{4.5355} = 1.764m$

∴ the line of action of the resultant cuts AB 1.764m from A

11. Given the equation $X^3 - 6x^2 + 9x + 2 = 0$
    (a) Show that the equation has a root between -1 and 0.

    Let f(x) = $X^3 - 6x^2 + 9x + 2$

    f(-1) = $(-1)^3 - 6(-1)^2 + 9(-1) + 2$

    = -1 – 6 -9 +2 = -14

    f(0) = 0 + 0 + 0 + 2

    = 2

    f(-1).f(0) = -14 x 2 = -28

    since f(-1).f(0) < 0; the root exist between -1 and 0.

    (b) (i) Show that the Newton Raphson formula approximating the root of the equation is given by $X_{n+1} = \frac{2}{3}\left[\frac{x_n^3-3x_n^2-1}{x_n^2-4x_n+3}\right]$

    f(x) = $X^3 - 6x^2 + 9x + 2$

    f(x_n) = $x_n^3 - 6x_n^2 + 9x_n + 2$

    f'(x_n) = $3x_n^2 - 12x_n + 9$

    $$x_{n+1} = x_n - \left(\frac{x_n^3-6x_n^2+9x_n+2}{3x_n^2-12x_n+9}\right)$$

    $$= \frac{x_n(3x_n^2-12x_n+9) - (x_n^3-6x_n^2+9x_n+2)}{3x_n^2-12x_n+9}$$

    $$= \frac{(3x_n^3-12x_n^2+9x_n) - (x_n^3-6x_n^2+9x_n+2)}{3x_n^2-12x_n+9}$$

    $$= \frac{2x_n^3-6x_n^2-2}{3x_n^2-12x_n+9}$$

    $$= \frac{2}{3}\left[\frac{x_n^3-3x_n^2-1}{x_n^2-4x_n+3}\right]$$

    (ii) Use the formula in (b)(i) above, with initial approximation of x_0 = -0.5, to find the root of the given equation correct to two decimal places

    Taking x = -0.5

    $x_1 = \frac{2}{3}\left[\frac{(-0.5)^3-3(-0.5)^2-1}{(-0.5)^2-4(-0.5)+3}\right]$ = -0.2381

    $|e| = |0.2381 - (-0.5)|$ = 0.2619

    $x_2 = \frac{2}{3}\left[\frac{(-0.2381)^3-3(-0.2381)^2-1}{(-0.2381)^2-4(-0.2381)+3}\right]$ = -0.1968

    $|e| = |0.1968 - (-0.2381)|$ = 0.0.0413

    $x_3 = \frac{2}{3}\left[\frac{-0.1968^3-3(-0.1968)^2-1}{(-0.1968)^2-4(-0.1968)+3}\right]$ = -0.1958

    $|e| = |-0.1958 - (-0.1968)|$ = 0.001< 0.005

    Hence the root = -0.20 (2D)

12. A newspaper vender by 12 copies of a sport magazine every week. The probability distribution for the number of copies sold in a week is given in the table below

Number of copies sold	9	10	11	12
probability	0.2	0.35	0.3	0.15

    (a) Estimate the

x	9	10	11	12
P(X = x)	0.2	0.35	0.30	0.15
xP(X = x)	1.8	3.5	3.3	1.8
x^2P(X = x)	16.2	35	36.3	21.6

(i) Expected number of copies that she sells in a week

Let x be the number of copies

E(X) = $\sum \mathrm{xP(X = x)}$= 1.8 + 3.5 + 3.3 + 1.8 = 10.4

(ii) Variance of the number of copies sold in a week.

Var (X) = $\sum \mathrm{x^2P(X = x)}$ - $(E(X))^2$

E(X^2) = $\sum \mathrm{x^2P(X = x)}$ = 16.2 + 35 + 36.3 + 21.6 = 109.1

Var (X) = 109.1 – $(10.4)^2$

= 109.1 – 108.16

= 0.94

(b) The vendor buys the magazine at shs. 1,200 and sells it at shs. 1,500. Any copies not sold are destroyed. Construct a probability distribution table for the vendor's weekly profit from the sales. Hence calculate her mean profit.

Profit for 9 copies = 9 x 1500 – 12 x 1200

= 13500 – 14400

= -900/=

Profit for 10 copies = 10 x 1500 – 12 x 1200

= 15000 – 14400

= 600/=

Profit for 11 copies = 11 x 1500 – 12 x 1200

= 16500 – 14400

= 2100/=

Profit for 12 copies = 12 x 1500 – 12 x 1200

= 18000 – 14400

= 3600/=

Note; the vendor buys 12 copies every week

Let y = weekly profit

Probability distribution is given by

y	-900	600	2100	3600
P(Y = y)	0.2	0.35	0.30	0.15
yP(Y = y)	-180	210	630	540

E(y) = $\sum \mathrm{yP(Y = y)}$ = -180 + 210 + 630 + 540 = 1200/=

Hence the expected weekly profit is shs. 1200/=

13. A particle starts from rest at a point (2, 0, 0) and moves such that its acceleration at any time t>0 is given by a = [16cos4ti + 8sin2tj + (sint – 2sin2t)k]ms^{-2}. Fins the

(a) speed when t = $\frac{\pi}{4}$.

a = [16cos4ti + 8sin2tj + (sint – 2sin2t)k]ms^{-2}

v = $\int adt$

= $\int [\mathrm{16cos4ti + 8sin2tj + (sint - 2sin2t)k}]\mathrm{dt}$

= [4sin4ti – 4cos2tj + (-cos t + cos2t)k]+ c

At t = 0

0 = [4sin0i – 4cos0j + (-cos 0 + cos0)k]+ c

0 = -4j + c

c = 4j

⇨ v = [4sin4ti + (-4cos2t + 4)j + (-cos t + cos2t)k

At t = $\frac{\pi}{4}$

⇨ v = [4sinπi + (-4cos$\frac{\pi}{2}$ + 4)j + (-cos $\frac{\pi}{4}$ + cos$\frac{\pi}{2}$)k

= 4j – cos $\frac{\pi}{4}$ k

$|v| = \sqrt{4^2 + \left(-\cos\frac{\pi}{4}\right)^2} = \sqrt{16 + \frac{2}{4}} = \sqrt{16.5}$ = 4.062ms^{-1}

(b) distance from the origin when t = $\frac{\pi}{4}$

s = $\int vdt$

=$\int [4\sin 4ti + (-4\cos 2t + 4)j + (-\cos t + \cos 2t)k$

=-cos4ti +(-2sin2t + 4t)j + (-sint + $\frac{1}{2}sin2t)k$ + c

At t = 0, s = 2i

By substitution

2i = -cos0 i +(-2sin0 + 4(0))j + (-sin0 + $\frac{1}{2}sin2\ (0))k$ + c

2i = -I + c

c = 3i

⇨ s = (-cos4t + 3)i +(-2sin2t + 4t)j + (-sint + $\frac{1}{2}sin2t)k$

At t = $\frac{\pi}{4}$

⇨ s = (-cosπ + 3)i +(-2sin$\frac{\pi}{2}$t + π)j + (-sin$\frac{\pi}{4}$ + $\frac{1}{2}sin\ \frac{\pi}{2})k$

= 4i + (π -2)j + $\left(-\frac{\sqrt{2}}{2} + \frac{1}{2}\right)k$

= 4i + 1.416j – 0.207k

$|s| = \sqrt{4^2 + (1.416)^2 + (-0.207)^2}$

= 4.24828

= 4.248 (3D)

14. The numbers x and y are approximate by X and Y with errors Δx and Δy respectively

(a) Show that the maximum relative error in xy is given by $\left|\frac{\Delta x}{x}\right| + \left|\frac{\Delta y}{y}\right|$.

Let z = xy and Z = XY

x =X + Δx and y = Y + Δy

z + Δz = (X + Δx)(Y + Δy)

= XY + XΔy + YΔx + ΔxΔY

Δz = XY + XΔy + YΔx + ΔxΔY – XY

= XΔy + YΔx + ΔxΔY

But Δx<< X, Δy << Y, => ΔxΔY = 0

⇨ Δz = XΔy + YΔx

R.E = $\left|\frac{\Delta z}{Z}\right|$

=$\left|\frac{X\Delta y}{XY} + \frac{Y\Delta x}{XY}\right|$

$= \left|\frac{\Delta y}{Y} + \frac{\Delta x}{X}\right|$

R.E $\leq \left|\frac{\Delta x}{X}\right| + \left|\frac{\Delta y}{Y}\right|$

R.Emax = $\left|\frac{\Delta x}{X}\right| + \left|\frac{\Delta y}{Y}\right|$

(b) If x = 4.95 and y = 2.013 are rounded off to the given number of decimal places, calculate

(i) percentage error in xy.

Percentage error = $\left[\left|\frac{\Delta x}{X}\right| + \left|\frac{\Delta y}{Y}\right|\right] x100\%$

X = 4.95, Δx= 0.005

Y = 2.013, Δy = 0.0005

Percentage error = $\left[\left|\frac{0.005}{4.95}\right| + \left|\frac{0.0005}{2.013}\right|\right] x100\%$= 0.126%

(ii) Limits within which xy is expected to lie, give your answer to three decimal places.

Minimum value = 4.945 x 2.0125 = 9.9518125 = 9.952 (3D)

Maximum value = 4.955 x 2.013 = 9.9768925 = 9.977 (3D)

Limits ar [9.952, 9.977]

15. The drying time of any newly manufactured paint is normally distributed with mean 110.5 minutes and standard deviation 12 minutes.

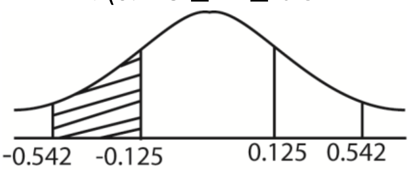
(a) Find the probability that paint dries between 104 and 109 minutes

Let x = drying time of the paint

P(104 $\leq x \leq 109$ = $P\left(\frac{104-110.5}{12} \leq z \leq \frac{109-110.5}{12}\right)$

= P(-0.542 $\leq z \leq -0.125$

= P(0.125 $\leq z \leq 0.542$



= P(0$\leq z \leq 0.542) - P(0 \leq z \leq 0.125$

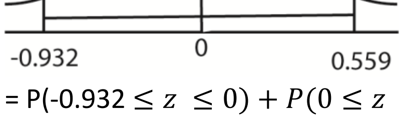
= 0.2061 – 0.0498

= 0.1563

(b) If a random sample of 20 tins of paint was taken, find the probability that the mean drying time of the samples is between 108 and 112 minutes.

P(108 $\leq \bar{X} \leq 112$) = $P\left(\frac{108-110.5}{\frac{12}{\sqrt{20}}} \leq z \leq \frac{112-110.5}{\frac{12}{\sqrt{20}}}\right)$

= P(-0.932 $\leq z \leq 0.559$



= P(-0.932 $\leq z \leq 0) + P(0 \leq z \leq 0.559)$

= 0.3243 + 0.2119

= 0.5362

16. A particle of mass 2kg moving with Simple Harmonic Motion (SHM) along the x-axis, is attracted towards the origin 0 by a force of 32x Newton. Initially the particle is at rest x = 20. Find the

(a) Amplitude and period of oscillation

ma = F

⇨ 2a = -32x

a = -16x

where $\omega^2 = 16$

$\omega = 4$

since the particle s momentarily at rest, when x = 20, the amplitude = A = 20m

Period $= \frac{2\pi}{\omega} = \frac{2\pi}{4} = \frac{\pi}{2} seconds$

(b) Velocity of the particle at any time, t>0

x = rcosωt

$v = \frac{dx}{dt} = \frac{d\,(20\cos 4t)}{dt}$ = -80sin4t

Hence velocity of the particle at any time, t is -80sin4t

(c) Speed when $t = \frac{\pi}{4}$ seconds

Speed $= |v| = \left|-80\sin\left(4\,x\,\frac{\pi}{4}\right)\right|$ = -80sinπ =0ms^{-1}