P425/1
PURE MATHEMATICS
PAPER 1
3 HOURS

UGANDA ADVANCED CERTIFICATE OF EDUCATION

POST MOCK SET 3 2020

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:
- Attempt **ALL** the **EIGHT** questions in section **A** and any **FIVE** from section **B**.
- All working must be clearly shown.
- Mathematical tables with list of formulae and squared paper are provided.
- Silent, non-programmable calculators should be used.
- State the degree of accuracy at the end of each answer using **CAL** for calculator and **TAB** for tables.
- Clearly indicate the questions you have attempted in a grid on your answer scripts.

Question		Mark
Section A		
Section B		
Total		

SECTION A (40 MARKS)

Answer all questions in this section.

1. In a triangle ABC, $\overline{AB} = x - y$, $\overline{BC} = x + y$, and $\overline{CA} = x$. Show that $\cos A = \frac{x - 4y}{2(x - y)}$. (5 marks)

2. If the line $3x - 4y - 12 = 0$ is the tangent to the circle with centre at (1,1). Find equation of the circle. (5 marks)

3. Using the substitution $u = \sqrt{x}$, show that $\int_1^4 \frac{dx}{x + \sqrt{x}} = In\left(\frac{9}{4}\right)$. (5 marks)

4. By using the Binomial theorem, expand $(25 + x)^{1/2}$ as far as the term in x^2. Hence evaluate $\sqrt{26}$ correct to 3 decimal places. (5 marks)

5. The points A, B, C and D have coordinates $(-7,9),(3,4),(1,2) and (-2,-9)$ respectively. Find the vector equation of the line PQ where P divides AB in the ratio 2:3 and Q divides CD in the ratio 1:-4. (5 marks)

6. A committee of four people is to be selected from a group of nine people. In how many ways can a committee be chosen if two particular people agree to serve only if both are selected? (5 marks)

7. If $y = ae^x + b\cos x$, show that;
$$(1 + \tan x)\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + (1 - \tan x)y = 0$$ (5 marks)

8. Find the volume generated if the area between the curve $y = 16 - x^2$, the y-axis and the line $y = 6x$ is rotated through four right angles about the x-axis. (5 marks)

SECTION B (60 MARKS)

Answer only five questions from this section.

9. (a) Given that $z_1 = 3 + 2i$ and $z_2 = 2 - i$. Find $z_1 + z_2$,graphically. (5 marks)

(b) If $z = x + iy$ is a complex number, describe and illustrate on the Argand diagram the locus of $\left|\frac{z+2}{z}\right| = 3$ (7 marks)

10. (a) Solve for θ, in the equation
$2\cos^2\left(\theta - \frac{\pi}{2}\right) - 3\cos\left(\theta - \frac{\pi}{2}\right) - 2 = 0$ where $0^0 \le \theta \le 360^0$ (5 marks)

(b) If $\tan\alpha = p$, $\tan\beta = q$, $\tan\gamma = r$, prove that $\tan(\alpha + \beta + \gamma) = \frac{p + q + r - pqr}{1 - pr - rq - pq}$,
hence, show that $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{\pi}{4}$.
(7 marks)

11. Find the values of A, B, C and D such that $\frac{2x^3 - 1}{x^2(2x-1)} = A + \frac{B}{x} + \frac{C}{x^2} + \frac{D}{(2x-1)}$.
Hence show that $\int_1^2 \frac{2x^3 - 1}{x^2(2x-1)}dx = \frac{3}{2} + In\left(\frac{4}{\sqrt{27}}\right)$. (12 marks)

12. (a) The points P and Q have position vectors $\underset{\sim}{\mathbf{p}}$ and $\underset{\sim}{\mathbf{q}}$ respectively with respect to an origin o. Point R lies on PQ and $\frac{PR}{PQ} = \lambda$, show that
$\underset{\sim}{\mathbf{r}} = (1-\lambda)\underset{\sim}{\mathbf{p}} + \lambda\underset{\sim}{\mathbf{q}}$ (4 marks)

(b) (i) Find the coordinate of the point P in which the plane $4x + 5y + 6z = 87$ intersects the line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z+4}{2}$

(ii) Determine the angle between the line and the plane in b (i)
(8 marks)

13. (a) If the line $y = 2x + c$ is a tangent to the hyperbola $\frac{x^2}{6} - \frac{y^2}{4} = 1$ show that $c = \pm 2\sqrt{5}$ (5 marks)

(b) A point P on the curve is given parametrically by $x = 3 - \cos\theta$ and $y = 2 + \sec\theta$. Find the equation of the normal to the curve at the point where $\theta = \pi/3$ (7 marks)

14. The first, fourth and eighth terms of an arithmetic progression (A.P) form a geometric progression (G.P). If the first term is 9, find the

(a) common difference of the AP (4 marks)

(b) common ratio of the G.P (2 marks)

(c) difference in sums of the first 6 terms of the progressions. (6 marks)

15. (a) Differentiate the following with respect to x and simplify your results.

(i) $y = 3^{xy}$

(ii) $y = \sin^4 x + \cos^4 x$ (6 marks)

(b) Find the coordinates of the turning point on the curve $y = \frac{\log_e x}{x}$ and identify the nature. (6 marks)

16. (a) Solve the differential equation

$x^2 \frac{dy}{dx} + xy = 2 + x^2$, given $y(1) = 2$ (5 marks)

(b) A body of unit mass falls under gravity in a medium in which the resistance R is proportional to the velocity, V of the body. If the body was initially at rest, show that the velocity after time t is given by, $V = gk^{-1}(1 - e^{-kt})$, where k is a constant and g is the acceleration due to gravity. (7 marks)

END