P425/1
PURE MATHEMATICS
PAPER 1
3 HOURS

UGANDA ADVANCED CERTIFICATE OF EDUCATION

POST MOCK SET 4 2020

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- Attempt **ALL** the **EIGHT** questions in section **A** and any **FIVE** from section **B**.
- All working must be clearly shown.
- Mathematical tables with list of formulae and squared paper are provided.
- Silent, non-programmable calculators should be used.
- State the degree of accuracy at the end of each answer using **CAL** for calculator and **TAB** for tables.
- Clearly indicate the questions you have attempted in a grid on your answer scripts.

<table>
<tr><td colspan="2">Question</td><td>Mark</td></tr>
<tr><td colspan="2">Section A</td><td></td></tr>
<tr><td rowspan="5">Section B</td><td></td><td></td></tr>
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SECTION A (40 MARKS)

1. Solve the simultaneous equations;
   $4^x = 6 \; and \; 2(2^x) = 3(2^y)$.

2. When a polynomial $P(x)$ is divided by $x^2 - 7x - 18$, the remainder is 2x + 5. Find the remainder when p(x) is divided by;
   (i) $x - 9$ (ii) $x + 2$

3. Show that; $\cot^{-1}\left(\frac{5}{2}\right) + \cot\left(\frac{9}{5}\right) = \cot^{-1}\left(\frac{35}{43}\right)$

4. Find the equation of the Normal to the circle $x^2 + y^2 - 2x - 24 = 0$ at the point (0, 7)

5. Evaluate; $\int_0^{\pi/3} \frac{\sin x}{(1+cosx)^2} \, dx$

6. The fourth, seventh and sixteenth terms of an A.P are in a G.P. If the first six terms of the A.P have a sum of 12. Find the common difference of the A.P and the common ratio of the G.P.

7. Find the position vector of the point of intersection, P, of the line with equation;
   $\boldsymbol{r} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and the line joining the points with position vectors $\begin{pmatrix} 2 \\ 4 \end{pmatrix} \; and \; \begin{pmatrix} 1 \\ 4 \end{pmatrix}$.

8. Find $\frac{dy}{dx} \; in \; y = In\left\{e^x \left(\frac{x-2}{x+2}\right)\right\}^{\frac{3}{4}}$

SECTION B

9. a) Expand up to the 4th term of
(i) $(1 + x)^{-1}$
(ii) $(1-2x)^{\frac{1}{2}}$, and hence show that if x is small than
$(1+x)^{-1} - (1-2x)^{\frac{1}{2}} \approx \frac{3x^2}{2}$ (6marks)
b) Apply the maclaraurin's series to establish a series for; $In\ (1+x)$. hence, if $x = \frac{b}{a}$.
Show that $\frac{b^2 - a^2}{2ab} = x - \frac{x^2}{2} + \frac{x^3}{2}$. (6marks)

10. a) Using the substitution $y = vx$, solve the differential equation.
$(2x-y)\frac{dy}{dx} = 2x + y$; given that $y = 3$ when $x = 2$. (5marks)
b) An electron survey revealed that during the parliamentary campaigns in a certain district, Mr. Katuntu was gaining support at a rate proportional to the product of the number of people already supporting him and those who were not yet supporting him. Po was the total population of the electorate and P was the number of people supporting him at a time t.
(i) Write a differential equation describing Mr. Katuntu's support.
(ii) If initially Mr. Katuntu had 100,000 supporters and the opinion polls revealed that he was gaining 5000 people per week, and the total electrorate being 2,000,000 people and that he is to win the election if he gets 51% of the vote, find how many weeks Mr. Katuntu needed to win the election.
(7marks)

11. (a) Find the acute angle between the line $\frac{x-6}{5} = \frac{1-y}{1} = z + 1$ and the plane,
7x – y + 5z = -5, giving your answer to the nearest degree.
(b) Find the point where the line $x + 1 = \frac{y-2}{4} = z - 3$ *cuts the plane* $\underset{\smile}{r} . \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = 8$
(6marks)

12. A curve is given by $y = \frac{x-1}{(2x-1)(x+1)}$. Sketch the curve clearly starting the asymptotes and the restricted region for y.
(12marks)

13. a) Show that 25x^2 + 9y^2 – 100x – 54y = 44. Represent an ellipse. State the co-ordinates of its;
(i) centre (ii) eccentricity (iii) Focii (7marks)
(b) A curve is given by the parametric equations X =4cos2t, y = 2sint. Show that they represent a parabola, state its vertex and sketch the curve.
(5marks)

14. Resolve $\frac{x^3-3}{(x-2)(x^2+1)}$ into partial fractions. Hence evaluate; $\int_3^4 \frac{x^3-3}{(x-2)(x^2+1)}\,dx$
(12marks)

15. a) Differentiate with respect to x;

(i) $(cosx)^{sinx}$

(ii) $tan^{-1}\left[\frac{1-x}{1+x}\right]dx$

(12marks)

Simplest form. (8marks)

b) If $y = e^{4x}cos3x; shat\ that\ \frac{dy^2}{dx^2} - 8\frac{dy}{dx} + 25y = 0$ (4marks)

16. a) Prove that in any triangle ABC

$$tanBcotC = \frac{a^2 + b^2 - c^2}{a^2 - b^2 + C}$$ (6marks)

(b) If $Z = (1 + cos2\theta) + i(\sin 2\theta); where\ \frac{\bar{\wedge}}{2} \leq \theta \leq \frac{\bar{\wedge}}{2}$ prove that $|Z| = 2cos\theta$.

(6marks)

END