P425/1
PURE MATHEMATICS
PAPER 1
3 HOURS

UGANDA ADVANCED CERTIFICATE OF EDUCATION

POST MOCK SET 7 2020

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- Attempt **ALL** the **EIGHT** questions in section **A** and any **FIVE** from section **B**.
- All working must be clearly shown.
- Mathematical tables with list of formulae and squared paper are provided.
- Silent, non-programmable calculators should be used.
- State the degree of accuracy at the end of each answer using **CAL** for calculator and **TAB** for tables.
- Clearly indicate the questions you have attempted in a grid on your answer scripts.

Question		Mark
Section A		
Section B		
Total		

SECTION A (40 MARKS)

1. If $y = e^x \sin x$, show that $\frac{d^2y}{dx^2} = 2\left(\frac{dy}{dx} - y\right)$.

(5marks)

2. Given that $\log_{10} 2 = a$, prove that $\log_8 5 = \frac{1-a}{3a}$. *(5marks)*

3. Solve by echelon method the following set of simultaneous equation:

$$x + 3y + z = 6$$
$$2x + y - 4z = 7$$
$$5x - 6y + z = 9$$

(5marks)

4. Partialise $f(x) = \frac{1}{(x+1)(x-3)}$ hence evaluate $\int f(x) \quad dx$. *(5marks)*

5. Form the equation of the circle which passes through A(1, 3)B(3, 5) and C(5, 3) in the form $x^2 + y^2 + 2fx + 2gy + c = 0$ hence state its centre and radius. *(5marks)*

6. Solve the equation $4\cos x - 2\cos 2x = 3$, for $0^0 \leq x \leq 2\pi$ *(5marks)*

7(i) In how many ways can the letters of the word GEOMETRY be arranged in a row?

(ii) In how many of these arrangements are the two E's together? *(5marks)*

8. The second, fourth and eighth terms of an AP are in a GP. The sum of the third and fifth terms is 20. Determine the first three terms of the AP stated above.

(5marks)

SECTION B (60 MARKS)

9a) If x is small enough so that terms in x^3 and higher powers may be ignored, use binomial expansion to show that $\sqrt{\left(\frac{1-x}{1+2x}\right)} = 1 - \frac{3x}{3} + \frac{15x^2}{8}$. ***(8marks)***

b) Expand by use of Maclaurin series up to the term in x^2 the function $f(x) = \sin x$. Hence evaluate $\sin 30^0$ to 4 decimal places. ***(4marks)***

10a) Initially, the number of bacteria present in a culture solution is N_o.
At time t = 1hour, the number of bacteria is measured to be $\frac{3N_0}{2}$. The rate of growth is assumed to be proportional to the number of bacteria present at any time t. Show that the time necessary for the bacteria to grow to $3N_o$ (triple the original) is approximately 2.7hrs. *(6marks)*

b) A small metal piece initially at 20^0C is dropped into a large container of water kept at 100^0C. It was observed that the temperature of the metal increased by 2^0C in one minute.

(i) How long will it take for the temperature of the metal to increase to 90^0C?

(ii) Find the temperature of metal after 20minutes. *(6marks)*

11. Evaluate the following:

(i) $\int_0^{\pi/4} x^2 \sin 3x \quad dx$ *(4marks)*

(ii) $\int \frac{x^3}{16+x^8}\,dx$ *(4marks)*

(iii) $\int \cos^5 x \quad dx$ *(4marks)*

12a)(i) Form the equation of the plane perpendicular to line $\frac{x-3}{2}=\frac{y+1}{-5}=\frac{z-4}{2}$ passing through a point **A**(5, -6, 6). *(4marks)*

(ii) Determine the point **B** where the formed plane meets the line in (i) above. *(4marks)*

b) Determine the shortest distance from the point P(2, -5, 3) to the line $\frac{x-1}{4}=\frac{y+3}{1}=\frac{z-2}{-2}$. *(4marks)*

13a) Prove that $\frac{\sin 3A\sin 6A+\sin A\sin 2A}{\sin 3A\cos 6A+\sin A\cos 2A}=\tan 5A$. *(4marks)*

b) Show that $\tan^{-1}\left(\frac{1}{3}\right)+\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)=\frac{\pi}{4}$. *(4marks)*

c) Solve for θ in the range $0 \leq \theta \leq 2\pi$ if $4\cos\theta + 3\sin\theta = 5$. *(4marks)*

14. Sketch the following curve systematically $y = \frac{3(x-3)}{(x+1)(x-2)}$. *(12marks)*

15a) Determine the square root of the complex number $15 + 8i$. *(4marks)*

b) Solve the equation $z^4 + 6z^2 + 25 = 0$ *(4marks)*

c) Evaluate $\frac{\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)^4}{\left(\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}\right)3}$ and give the solution in modulus-argument form.

(4marks)

END