

P425/1
PURE MATHEMATICS
PAPER 1
3 HOURS

UGANDA ADVANCED CERTIFICATE OF EDUCATION

PRE- MOCK SET 10 2020

PURE MATHEMATICS

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- Attempt **ALL** the **EIGHT** questions in section **A** and any **FIVE** from section **B**.
- All working must be clearly shown.
- Mathematical tables with list of formulae and squared paper are provided.
- Silent, non-programmable calculators should be used.
- State the degree of accuracy at the end of each answer using **CAL** for calculator and **TAB** for tables.
- Clearly indicate the questions you have attempted in a grid on your answer scripts.

Question		Mark
Section A		
Section B		
Total		

SECTION A: (40 MARKS)

Answer **all** questions in this Section.

1. Solve the simultaneous equations

$$\begin{aligned}xy &= 2 \\ 2\log(x-1) &= \log y\end{aligned}\quad (05 \text{ marks})$$

2. Differentiate $y = \frac{x-2}{\sqrt{1-x^2}}$ with respect to x . (05 marks)

3. Without using tables or calculators, show that $\tan^2 22.5^\circ = 3 - 2\sqrt{2}$. (05 marks)

4. Find the gradients of the two tangents from the point $(3, -2)$ to the circle $x^2 + y^2 = 4$. (05 marks)

5. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin x}{3 + 5\cos x} dx$. (05 marks)

6. The points A and B have position vectors $4\mathbf{i} + 3\mathbf{j}$ and $\mathbf{i} + t\mathbf{j}$. Determine the values of t such that the angle $\angle AOB = \cos^{-1} \frac{2}{\sqrt{5}}$, where O is the origin. (05 marks)

7. A hemispherical bowl of internal radius 15 cm contains water to a depth of 7 cm, find the volume of the water in the bowl correct to 1 decimal place. (05 marks)

8. Compute the sum of four- digit numbers formed with the four digits 2, 5, 3, 8 if each digit is used only once in each arrangement. (05 marks)

SECTION B: (60 MARKS)

Answer **only** five questions. All questions carry equal marks.

9. (a) Show that the curve $x = 5 - 6y + y^2$ represents a parabola and find the directrix. (05 marks)
- (b) (i) Find the equation of the chord through the points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ of the parabola $y^2 = 4ax$.
(ii) Show that the chord in (b) (i) cuts the directrix where $y = \frac{2a(pq - 1)}{p + q}$. (07 marks)
10. (a) The roots of the equation $2x^2 - 3x + 5 = 0$ are α and β . Find the equation whose roots are $\frac{\alpha}{\beta - 2}$ and $\frac{\beta}{\alpha - 2}$. (06 marks)
- (b) Solve the equation $\sqrt{\frac{x - 1}{3x + 2}} + 2\sqrt{\frac{3x + 2}{x - 1}} = 3$. (06 marks)
11. (a) Given that $x = \sec A - \tan A$, prove that $\tan \frac{A}{2} = \frac{1 - x}{1 + x}$. (05 marks)
- (b) Solve the equation $\sin t \cos 3t + \sin 3t \cos t = 0.8$ for $0 \leq t \leq 2\pi$. (05 marks)
12. Given that $y = \sin(2\sin^{-1} x)$, prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$. Hence, by Maclaurin's theorem, expand y as far as the term in x^3 . (12 marks)
13. The position vectors of points P and Q are $2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ and $3\mathbf{i} - 7\mathbf{j} + 12\mathbf{k}$ respectively.
- (a) Determine the length of $\mathbf{PQ}$. (03 marks)
- (b) Given that the line PQ meets the plane $4x + 5y - 2z = 5$ at the

point T. Find the:

- (i) co-ordinates of T,
- (ii) angle between line PQ and the plane. (09 marks)

14. (a) Express $y = \frac{x^4 + 2x}{(x-1)(x^2+1)}$ into partial fractions. **Turn Over**
(07 marks)

(b) Evaluate $\int_2^4 y \, dx$. (05 marks)

15. (a) Show that $2 - 3i$ is a root to the equation.

$$z^4 - 5z^3 + 18z^2 - 17z + 13 = 0.$$

Hence find the other roots of the equation. (06 marks)

- (b) Using De Moivre's theorem, find the cube roots of $-4 + 6i$. (06 marks)

16. (a) Solve the differential equation $\frac{dr}{d\theta} + 2r \tan \theta = \sec^2 \theta$. (05 marks)

- (b) The tangent at any point $P(x, y)$ on the curve, cuts the x-axis at A and the y-axis at B. Given that $2\mathbf{AP} = \mathbf{PB}$ and that the curve passes through the point $(1, 1)$, find the equation of the curve. (07 marks)

END